Modelling Endogenous Interbank Networks

Grzegorz Hałaj   Christoffer Kok

European Central Bank

July 4, 2013

Motivation

- Recent financial crisis: loss of trust on the interbank market; concerns about failure of one of the key players spreading contagion; small shocks with detrimental effects
- A response from regulators: measures to mitigate the risk ⇒ higher capital standards + reducing bilateral exposures
  - Large Exposure (LE) limits;
  - Credit Valuation Adjustment (CVA) to unlock the risk in OTC exposures and immediately reflect it in the capital
  - standard settlement practices (CCP framework)
- Our aim:
  - fill the gap in the literature to improve understanding of risk stemming from interconnectedness
  - impact of different policy / regulatory measures on the interbank structure
- Approach: modelling of banks' reactions to these measures and to the changing macroeconomic environment (risk and return potential on the interbank market, funding conditions...)

Outline

Modeling framework

- Four round model
  1. offers of interbank placements based on individual optimisation of interbank asset structures
  2. funding diversification
  3. negotiation phase: matching offers and preferred funding structure in a bargaining game
  4. price (i.e. interest rate) adjustment

Scope for application

- impact of CVA estimates
- parametrisation of LE limits
- stress tests and dynamic balance sheet tool – passing macro scenarios via a dynamic balance sheet model to obtain changes in the aggregate interbank lending

Literature – towards network formation

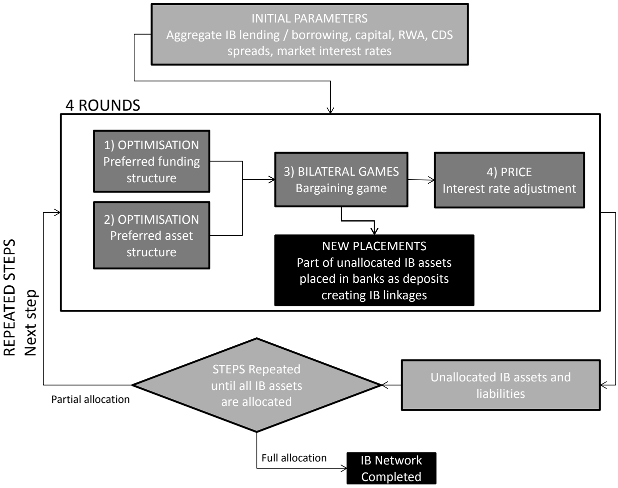
- Interbank market may (in normal times) act as a shock absorber and peer monitoring mechanism (see e.g. Bhattacharya and Gale, 1997; Flannery, 1996; Rochet and Tirole, 1996)
- But interbank market can also be a source of contagion (Allen and Gale, 2000; Nier et al., 2008; Allen and Babus, 2009)
- Static networks – estimated (e.g. Upper and Worms, 2004 and many more), modelled with mechanistic rules (e.g. Nier et al., 2007 and many more) ⇒ properties studied by means of various network measures or shock propagation methods (e.g. CPV of Eisenberg and Noe (2001))
- Networks in other research areas: game theory of Jackson and Wolinsky (1996)
- Extensions in finance – exogenous networks: game theory – optimal responses of banks to shocks to incentives to lend Cohen-Cole (2011) Bluhm 2013. Acemoglu (2013): dealing with social inefficiency of financial networks; Georg (2011) models interbank exposures as residuals of banks' investment activities (but networks simply drawn from a distribution)
- Castiglionesi (2011) propose Nash equilibrium of networks maximising the investors payoff which depends of the interbank neighbourhood of banks in which an investor allocated funds (network independent of banks' decisions)
- Agent-based approach to address overly complex equilibria – Markose (2012); Grasselli (2013)
- Portfolio choice (Markowitz 1952), matching (Chen 2013) and price formation (Eisenschmidt 2009) ⇒ mechanisms important for us

Endogenous networks

The aim of the study is to:

1. understand the foundations of the topology of the interbank networks
2. analyse sensitivity of the interbank network structures to banks' heterogeneity (in terms of size of balance sheet, capital position, general profitability, counterparty credit risk) and the changes of market and bank specific risk parameters, embedding optimising behaviour of banks
3. project the evolution of the interbank structure (given a macro scenario)
4. assess effectiveness of rule designed to mitigate systemic risk on the interbank system (esp. pertaining to capital requirements, size and diversity of interbank exposures)

Figure 1 : The sequential four round procedure of the interbank formation



Prerequisites

- (banks) N banks in the system
- (interbank exposures) Let L_{ij} denotes the interbank placement of bank j in bank i.
- (capital position) total capital e and capital $e^I \leq e$ allocated to the interbank assets; risk weights ω of interbank exposures.
- (CVA) Exposure L_{ij} requires to deduct $\gamma_i L_{ij}$ from capital e^I_j, for γ_i being a bank-specific CVA factor, to account for the market based assessment of the credit risk related with bank i.
- (probability map P) of interbank connections drawn from P allowing for capturing possible customer relationship between banks. Each bank j draws its counterparties $B^k_j \subset \mathbb{N}/\{j\}$, enlarging the set at each step k: $\bar{B}^{k+1}_j = \bar{B}^k_j \cup B^{k+1}_j$.
- (matching) at each step k incremental matching of assets and liabilities: $\bar{a}^k_j = \bar{a}^{k-1}_j - \sum_i L^k_{ij}$, where L^k is a matrix of placements at step k

CVA calibration

We follow the advanced approach philosophy to capture the sensitivity of additional capital requirements to banks' individual credit spreads

$$\mathrm{CVA}_i = \lambda \sum_{n \leq N} \Big(\exp(-s_i^n t^n / \lambda) - \exp(-s_i^{n-1} t^{n-1} / \lambda) \Big) \frac{EE^n d_i^n + EE^{n-1} d_i^{n-1}}{2},$$

where

- λ is the market based assessment of LGD on the exposure against a given counterpart,
- t^n is the n-th point of time,
- s_i^n is the projected CDS spread of bank i at time n,
- EE^n is the projected, netted exposure against the counterpart,
- d_i^n is a discount factor (bank or rather country specific).

Assumption: $EE^n = EE^*$ for all n then $\gamma_i := \mathrm{CVA}/EE^*$ is a proportionality parameter

1^{st} round – Criteria for investment of interbank assets

General idea of banks' optimising behaviour

Assumption: each bank maximises return from interbank portfolio adjusted by risk related to interest rates and counterparts' defaults (with a predefined risk aversion parameter) and taking into account customer relationship, i.e. a drawn sample of banks

Each bank maximises the following function of its interbank exposure breakdown:

$$J(L_{1j}, \ldots, L_{Nj}) = \sum_{i \in \bar{B}_j^k} r_i L_{ij} - \kappa_j (\sigma * L_{\cdot j}^\top)^\top Q (\sigma * L_{\cdot j}) \tag{1}$$

Outcome: a matrix of exposures $\mathbf{L}^{\mathbf{I},\mathbf{k}}$, whereby optimisation subject to constraints...

...Constraints of the admissible set of strategies

The maximisation is subject to some feasibility and capital constraints.

1. budget constraint – $\sum_{j|j\neq i} L_{ij} = \bar{a}_j^k$ and $L_{jj} = 0$, for $a_j^0 = a_j$ being exogenously determined;
2. counterpart's size constraint – $L_{ij} \leq \bar{l}_i^k$;
3. capital constraint – $\sum_{i|i\neq j} \omega_i (L_{ij}^k + L_{ij}) \leq e_j^I - \gamma^\top (\bar{L}_{\cdot j} + L_{\cdot j})$;
4. large exposure limit constraint – $L_{ij} \leq \chi e_j$.

What if the constraints are too stringent for a bank j? $\Rightarrow$ bank j reduces its interbank lending and (technically) the optimisation is solved for $\bar{a}_j^k$ replaced by $\bar{a}_i^k - 2\Delta\bar{a}_i^k$, $\bar{a}_i^k - 3\Delta\bar{a}_i^k$,... until $\bar{a}_i^k - k_i\Delta\bar{a}_i^k$ gives a feasible set of constraints

2^{nd} round – funding diversification

Diversification risk gauged by default risk

$$X_j := \begin{cases} 0 & \text{with probability } p_j \\ 1 & \text{with probability } 1 - p_j \end{cases} \quad (2)$$

Remark: p_js are risky (variance based on observed time series of CDS spreads)
For a covariance matrix $\bar{D}^2_X$ of X and funding risk aversion κ^F the optimised funding risk is measured

$$F(L^k_{i1}, \ldots, L^k_{iN}) = \kappa^F [L^k_{i1} \ \ldots \ L^k_{iN}] \bar{D}^2_X [L^k_{i1} \ \ldots \ L^k_{iN}]^\top \quad (3)$$

Outcome: a matrix of interbank deposits $\mathbf{L^{F,k}}$, whereby optimisation on the admissible set:
$\mathcal{A}^F_i := \{y \in \mathbb{R}^N_+ | j \in \bar{B}^k_j \Rightarrow y_j \leq \bar{a}^k_j \text{ and } j \notin \bar{B}^k_j \Rightarrow y_j = 0\}$.

3^{rd} round – the game

Assumption: banks negotiate in pairs simultaneously (pair (i', j) knows the outcome of (i'', j) after both games are completed). Case $L_{ij}^{I,k} > L_{ij}^{F,k}$

$$G_{ij}^{k}(x) = \left[U_{ij}^{I,k*} - s_{ij}^{I,k} \cdot (x - L_{ij}^{F,k})\right]\left[U_{ij}^{a,k*} - s_{ij}^{a,k} \cdot (L_{ij}^{I,k} - x)\right] \qquad (4)$$

where $s_{ij}^{I,k}$ is a measure of how much bank i is willing to deviate from his optimal funding strategy, i.e.

$$s_{ij}^{I,k} = \max\left(\frac{U_{ij}^{I,k}(L_{ij}^{F,k}) - U_{ij}^{I,k}(L_{ij}^{I,k})}{|L_{ij}^{I,k} - L_{ij}^{F,k}|}, 0\right),$$

where $U_{ij}^{I,k}(x) = -F(L_{i1}^{F,k}, \dots, L_{ij-1}^{F,k}, x, L_{ij+1}^{F,k}, \dots, L_{iN}^{F,k})$
and $U_{ij}^{I,k*}$ and $U_{ij}^{a,k*}$ are utilities at the individual optima
(for $s_{ij}^{a,k}$ analogously,... and for $L_{ij}^{I,k} < L_{ij}^{F,k}$ similar)
Goal of the game: maximisation of G_{ij}^{k}

4^{th} round – price adjustment

- After the first 3 rounds of a step k some banks may still have a gap in the interbank funding $\Rightarrow$ adjustment to the offered interest rate on new interbank deposits to increase a chance to obtain funding in step $k+1$
- If at the step $k+1$ the gap amounts to $g_i^{k+1} := l_i - \sum_j \bar{L}_{ij}^{k+1}$ then the offered rate $r_i^{k+1} = r_i^k \exp(\alpha g_i^{k+1}/l_i)$
- α is price elasticity parameter

Data

- subsample of EBA sample 2011
- Bankscope data on TA_i, interbank borrowing and lending (l and a), customer loans L_i, securities S_i and capital e_i
- probability map based on 2011 EBA disclosures on geographical breakdown of banks' portfolios related to the capital exercise
- risk weights for interbank lending are set to 20%, loans and securities are calibrated assuming that credit risk related RWAs are solely to loan exposures
- in this way:

$$e_i^I = \frac{20\% a_i}{20\% a_i + \mathrm{RW}^L L_i + \mathrm{RW}^S S_i} e_i$$

Sensitivity analysis

How sensitive are the results to the key parameters

several parameters are chosen:

- correlation
- investment and funding risk aversion
- Loss Given Default
- Price adjustment
- Capital allocation to the interbank portfolio

Does the model perform well?

To verify:

- (correlation) decreasing correlation $\Rightarrow$ higher potential for diversification of risk in interbank assets $\Rightarrow$ seeking more counterparts
- (funding) lower propensity of banks to accepts larger placements $\Rightarrow$ higher connectivity

Table 1 : Sensitivity

Parameter (baseline)	Tested value	Network measure – avg. weighted change vs baseline (%)				
		Out-deg	In-deg	Bness	Katz	DebtRank
correlation (estim.)						
	0 corr.	-36.2	-35.2	-7.3	-14.0	-7.6
		(1.8)	*(0.8)*	*(0.1)*	*(0.9)*	*(0.9)*
LGD (0.4)						
	0.3	-1.6	0.2	1.3	1.5	2.3
		(0.1)	*(0.4)*	*(0.0)*	*(0.3)*	*(0.1)*
	0.5	1.8	-0.8	-0.5	-1.4	-0.1
		(0.2)	*(0.3)*	*(0.1)*	*(0.1)*	*(0.0)*
Inv. risk aversion (0.5)						
	1.5	-3.0	-1.0	0.2	0.5	0.9
		(0.1)	*(0.2)*	*(0.5)*	*(0.1)*	*(0.3)*
	2.5	-3.7	-1.0	-4.2	0.7	1.2
		(0.1)	*(0.1)*	*(0.7)*	*(0.2)*	*(0.2)*
Fund. risk aversion (0.5)						
	1.5	1.2	0.1	3.4	5.2	3.8
		(0.5)	*(0.0)*	*(0.5)*	*(0.8)*	*(0.6)*
	2.5	2.3	0.1	5.6	5.6	3.7
		(0.7)	*(0.0)*	*(0.6)*	*(0.7)*	*(0.1)*
Price elast. (0.0)						
	0.5	4.1	5.2	10.3	-0.6	-1.8
		(1.7)	*(0.6)*	*(2.4)*	*(0.3)*	*(0.1)*
	1.0	4.4	-2.3	12.8	3.2	-1.7
		(0.5)	*(0.4)*	*(1.9)*	*(0.3)*	*(2.1)*
Alloc. capital (el)						
	$0.9el$	2.6	0.6	10.0	-4.2	-3.7
		(0.2)	*(0.3)*	*(0.6)*	*(0.7)*	*(0.8)*
	$1.1el$	5.9	3.4	3.7	-1.6	-2.7
		(0.3)	*(0.3)*	*(0.6)*	*(0.6)*	*(0.5)*

Applications – policy implications

Large Exposure limits – compactness of the networks

lower bilateral exposures allowed ⇒ more connections

CVA – crowding out bad quality borrowers

supposedly, banks would shift towards lending to high quality borrowers

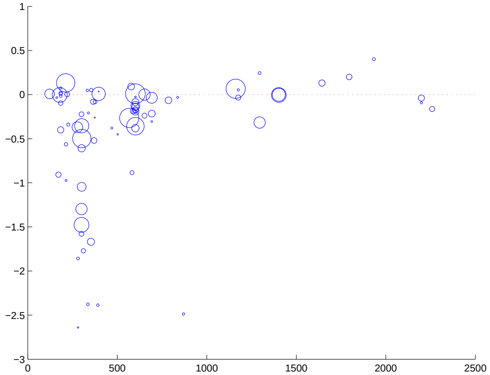
Network reactions to adverse market conditions

passing macro scenarios via dynamic BS model (Hałaj, 2013) to get new optimal aggregate interbank lending / borrowing and capitalisation:

baseline macro scenario ⇒ optimising behaviour of banks to change the structure of assets in order to maximise risk-adjusted return ⇒ change in banks' preferred aggregate interbank lending and borrowing ⇒ endogenous formation of the interbank under specified regulator regime ⇒ adverse macro shock ⇒ banks defaults ⇒ contagion spreading across the network (clearing payments vector)

- x-axis: CDS spread (in bps)
- y-axis: difference of CAR after adverse stress testing shock between LE=15% and LE=25% regime (in pp, positive number means that by lowering LE limit contagion losses rise)
- The size of a circle is proportional to bank's total assets
- Stringent LE limits overall tend to lower contagion risk
- Benefits for the safest part of the banking system

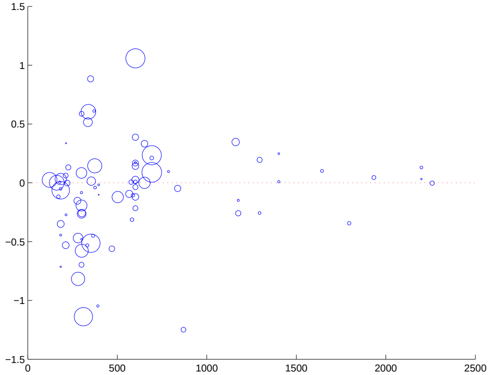
Figure 2 : Counterparty credit quality and the impact of LE limits on the losses incurred due to contagion



Source: own calculations

- x-axis: CDS spread (in bps)
- y-axis: difference of CAR after adverse stress testing shock between CVA and no CVA regime (in pp, positive number means that introduction of CVA charge increases contagion losses).
- The size of a circle is proportional to bank's total assets.
- Similarly, the impact of the CVA on contagion risk is mainly affecting the sounder banks.
- Direction of the impact is however ambiguous.

Figure 3 : Counterparty credit quality and the impact of CVA capital charge on the losses incurred due to contagion



Source: own calculations

Conclusions

- Endogenous interbank networks give an important insight into the role of banks' investment and funding strategies in shaping the interbank market. The simple, mechanistic cascade models are too simplistic in assuming that banks do not react to actions of other interbank participants and market conditions.
- In the proposed framework, we are able to analyse different policy measures addressing the systemic risk – their ultimate impact on the market structure and efficiency in reducing the contagion risk.
- More stability and robustness checks must be performed in order to understand the complexity of the relationship between market parameters and network topologies.
- The model needs to be calibrated to the observed interbank networks; especially with regard to the probability map and CVA.