Macroeconomic instability and microeconomics financial fragility: a stock-flow consistent approach with heterogeneous agents[1]

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Interlinkages and Systemic Risk
Ancona, July 5 2013

[1]Financial support by INET is gratefully acknowledged

- Features:
  - This paper introduces heterogeneous microeconomic behavior into a demand-driven SFC model.
  - Analytical aggregation of heterogeneous agents by means of statistical mechanics tools (Aoki 2006, Di Guilmi 2008, Foley JET 1994, Landini and Uberti CE 2008).
- Objectives:
  - present the analytical links between the **financial micro-variables** and the macroeconomy;
  - assess the effects of the interaction between leverage dynamics, financialization and income inequality.

Stock-flow consistent modeling

Table 2.6 Transactions flow matrix

	Households	Production firms		Banks		Government	Central Bank		
	(1)	Current (2)	Capital (3)	Current (4)	Capital (5)	(6)	Current (7)	Capital (8)	Σ
Consumption	$-C$	$+C$							0
Investment	$-I_h$	$+I$	$-I_f$						0
Govt. exp.		$+G$				$-G$			0
Wages	$+WB$	$-WB$							0
Profits, firms	$+FD_f$	$-F_f$	$+FU_f$						0
Profits, banks	$+FD_b$			$-F_b$	$+FU_b$				0
Profit, central Bk						$+F_{cb}$	$-F_{cb}$		0
Loan interests	$-r_{l(-1)} \cdot L_{h(-1)}$	$-r_{l(-1)} \cdot L_{f(-1)}$		$+r_{l(-1)} \cdot L_{(-1)}$					0
Deposit interests	$+r_{m(-1)} \cdot M_{h(-1)}$			$-r_{m(-1)} \cdot M_{(-1)}$					0
Bill interests	$+r_{b(-1)} \cdot B_{h(-1)}$			$+r_{b(-1)} \cdot B_{b(-1)}$		$-r_{b(-1)} \cdot B_{(-1)}$	$+r_{b(-1)} \cdot B_{cb(-1)}$		0
Taxes – transfers	$-T_h$	$-T_f$		$-T_b$		$+T$			0
Change in loans	$+\Delta L_h$		$+\Delta L_f$		$-\Delta L$				0
Change in cash	$-\Delta H_h$				$-\Delta H_b$			$+\Delta H$	0
Change, deposits	$-\Delta M_h$				$+\Delta M$				0
Change in bills	$-\Delta B_h$				$-\Delta B_h$	$+\Delta B$		$-\Delta B_{cb}$	0
Change, equities	$-(\Delta e_f \cdot p_{ef} + \Delta e_b \cdot p_{eb})$		$+\Delta e_f \cdot p_{ef}$		$+\Delta e_b \cdot p_{eb}$				0
Σ	0	0	0	0	0	0	0	0	0

- Traditional aggregative SFC models describes the interlinkages among sectors but excludes those among agents and the role of heterogeneity.
- Minsky (1975): "*an ultimate reality in a capitalist economy is the set of interrelated balance sheets among the various units*".
- This project aims to represent the macroeconomic effects of the joint dynamics of firms' balance sheet variables.

Steps of the analysis

1. Set up and numerically simulate the ABM with N heterogeneous firms belonging to two different categories (borrowing and hedge);
2. identifying a *representative* firm within each group (*mean-field approximation*) by taking the average of firms' variables;
3. set up and solve the master equation (ME) to obtain the dynamics of the number of firms in each group;
4. build the SFC system with the solution of the ME and the dynamic equations for the macro-variables.

- Two types of firms
  - **borrowing firms** ($\mathbf{z = 1}$): cannot finance all investment with internal finance and issue equities and bonds;
  - **hedge firms** ($\mathbf{z = 2}$): finance all investment with retained profits.
- investment function:

$$i_z^j(t) = \alpha\ h(t) + \beta_z\ a^j(t) + \epsilon\ u^j(t) \tag{1}$$

  i: investment; a: retained profits; u: capacity utilization ratio; $h(t) = Pe(t)E(t)/P\ K(t)$, Pe: stock price; K: aggregate capital; $\alpha, \beta_z, \epsilon > 0$.
- Fazzari et al. 1988: $\beta_1 > \beta_2$;
- for borrowing firms, $i - a$ is financed for a fraction ϖ with equities and the rest with debt.

- the quantity actually sold by a firm is a fraction of total demand proportional to its size plus a uniformly distributed *preferential attachment* shock s with $\mathbb{E}[s] = 0$;
- price: constant mark-up μ on the cost of labor;
- accordingly, output shares of labor (Ψ) and profit (Π) share of output are given exogenously;
- A firm fails if $a^j/K^j \leq c$, where c is a constant.

- The propensities to save out of workers' and managers' disposable income (s_Ψ and s_Θ) and out of managers' capital gains (σ_Θ) are constant parameters.
- wealth accumulates for savings S and capital gain G;
- households allocate their wealth W between money M and shares E;
- the demand for firms' shares depends on previous capital gains:

$$\frac{Pe(t)E(t)}{W(t)} = \frac{1}{1 - exp[\lambda_r r - \lambda_G G(t - dt)]} \tag{2}$$

- the interest rate for deposits and loans is the same.

Stochastic evolution of the two types of firms

1. Quantify the transition probabilities η and ζ as functions of the preferential attachment shock s and the firms' balance sheets variables;
2. define the ME:

$$\frac{dP(N_1,t)}{dt} = \text{(probability of flow of firms into the borrowing state) - (probability of flow of firms out of the borrowing state)}$$

3. the solution of the ME yields the dynamics of n_1:

$$\dot{n_1}(t) = \eta m(t) - (\eta + \zeta)[m(t)]^2 + \sigma \, dV(t) \qquad (3)$$

m:trend; dV is a stationary Wiener increment and $\sigma = \frac{\eta\zeta}{(\eta+\zeta)^2}$.

- Solution of the ME: dynamics of N_1;
- total investment:

$$\dot{K}(t) = I(t) = N_1 i_1 + N_2 i_2 = N\alpha h(t) + N_1[\beta_1 a_1(t) + \epsilon u_1(t)] + N_2[\beta_2 a_2(t) + \epsilon u_2(t)] \quad (4)$$

- dynamics of aggregate debt:

$$\dot{B}(t) = N_1 \{\varpi [i_1(t) - a(t) - m_1(t)]\} \quad (5)$$

- total number of shares:

$$\dot{E}(t) = N_1 \left\{(1-\varpi)\left[i_1^j(t) - a^j(t) - m_1^j(t)\right]/Pe(t)\right\} \quad (6)$$

	Households	Firms		Banks	Total
		Current	Capital		
Consumption	$-C$	$+C$			0
Investment		$+I$	$-[N\alpha h + N_1(\beta_1 a_1 + \epsilon u_1) + N_2(\beta_2 a_2 + \epsilon u_2)]$		0
Wages	$+\Psi pQ$	$-\Psi pQ$			0
Profits	$+\Theta(\Pi pQ - rB)$	$-(N_1 a_1 + N_2 a_2)$	$(1-\Theta)(\Pi pQ - rB) + r(N_1 m_1 + N_2 m_2)$		0
Loan interests		$-rB$		rB	0
Deposit interests	$r(M_\Psi + M_\Theta)$	$+r(N_1 m_1 + N_2 m_2)$		$-r(M_\Psi + M_\Theta + N_1 m_1 + N_2 m_2)$	0
Change in loans			$+N_1[\bar{\omega}(i_1 - a - m_1)]$	$-\dot{B}$	0
Change in deposits	$-(\dot{M}_\Psi + \dot{M}_\Theta)$		$-(N_1 \dot{m}_1 + N_2 \dot{m}_2$	$\dot{M}$	0
Change in equities	$-Pe\dot{E}$		$+N_1[(1-\bar{\omega})(i_1 - a - m_1)]$		0
Total	0	0	0	0	0
Capital Gains	$-\dot{P}eE$		$+\dot{P}eE$		0

Figure: Matrix of flows

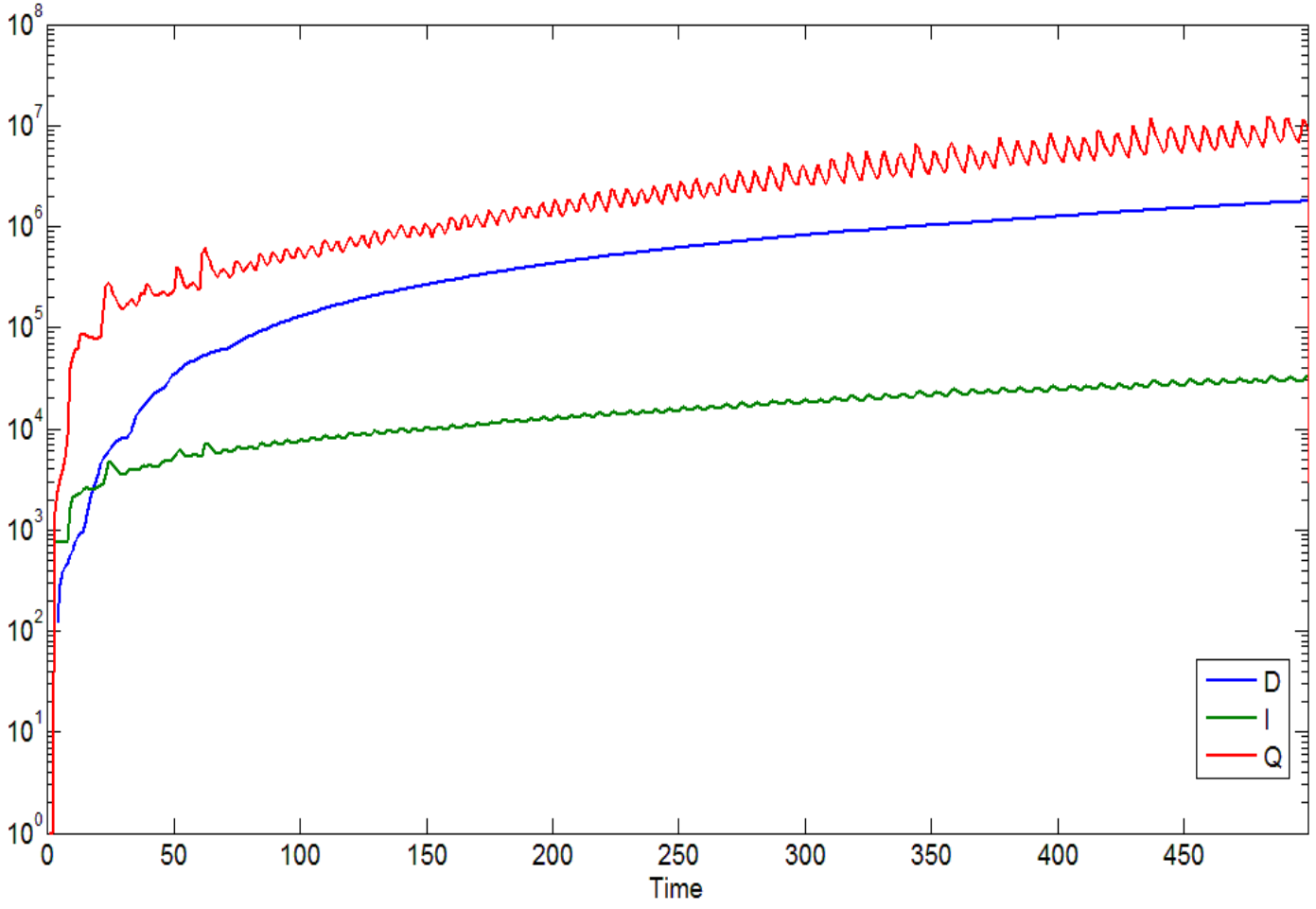


Figure: Dynamics of aggregate demand, investment and debt.

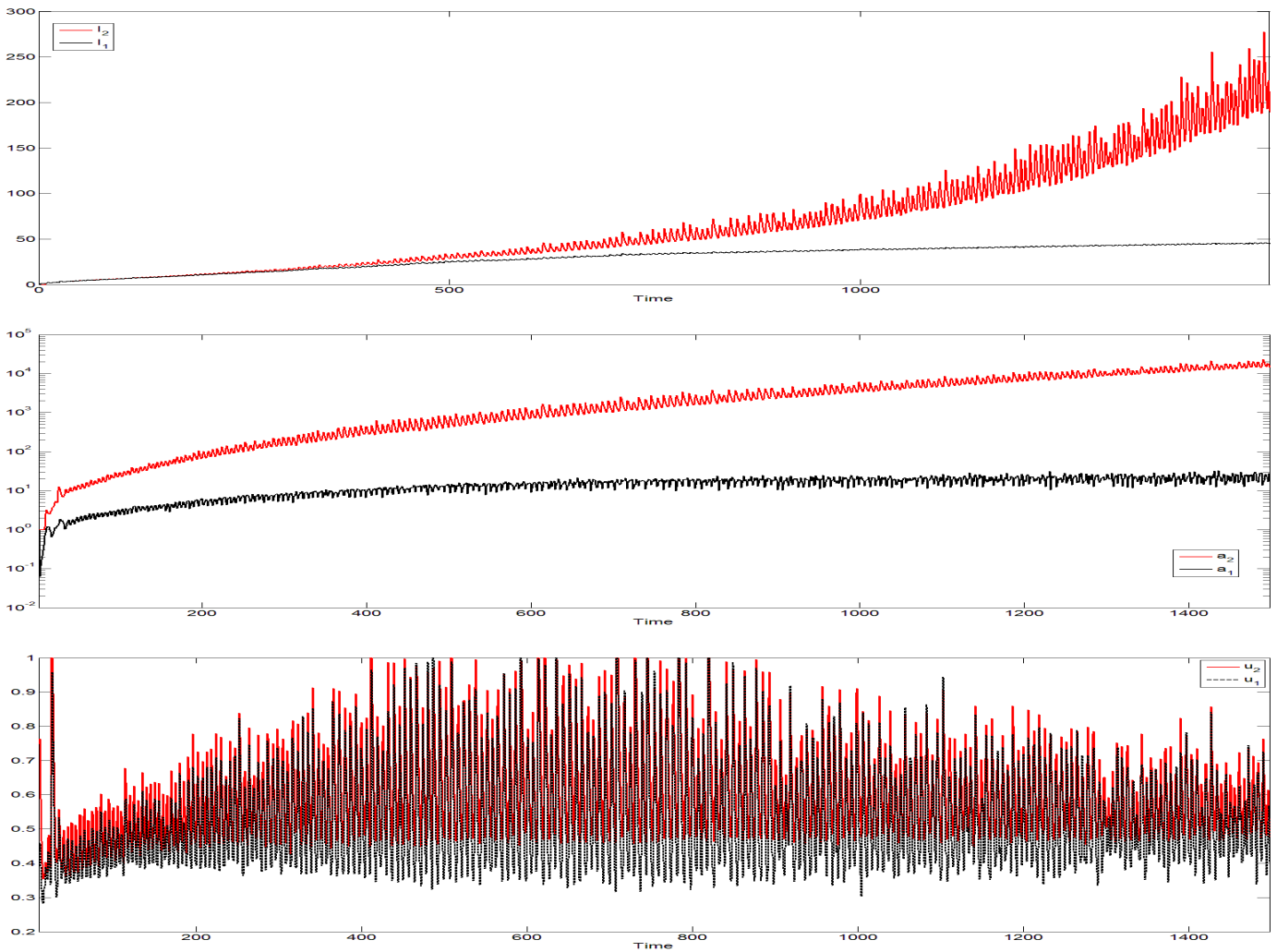


Figure: Investment (top), accumulated profit (center) and capacity utilization (bottom) for average borrowing and hedge firms.

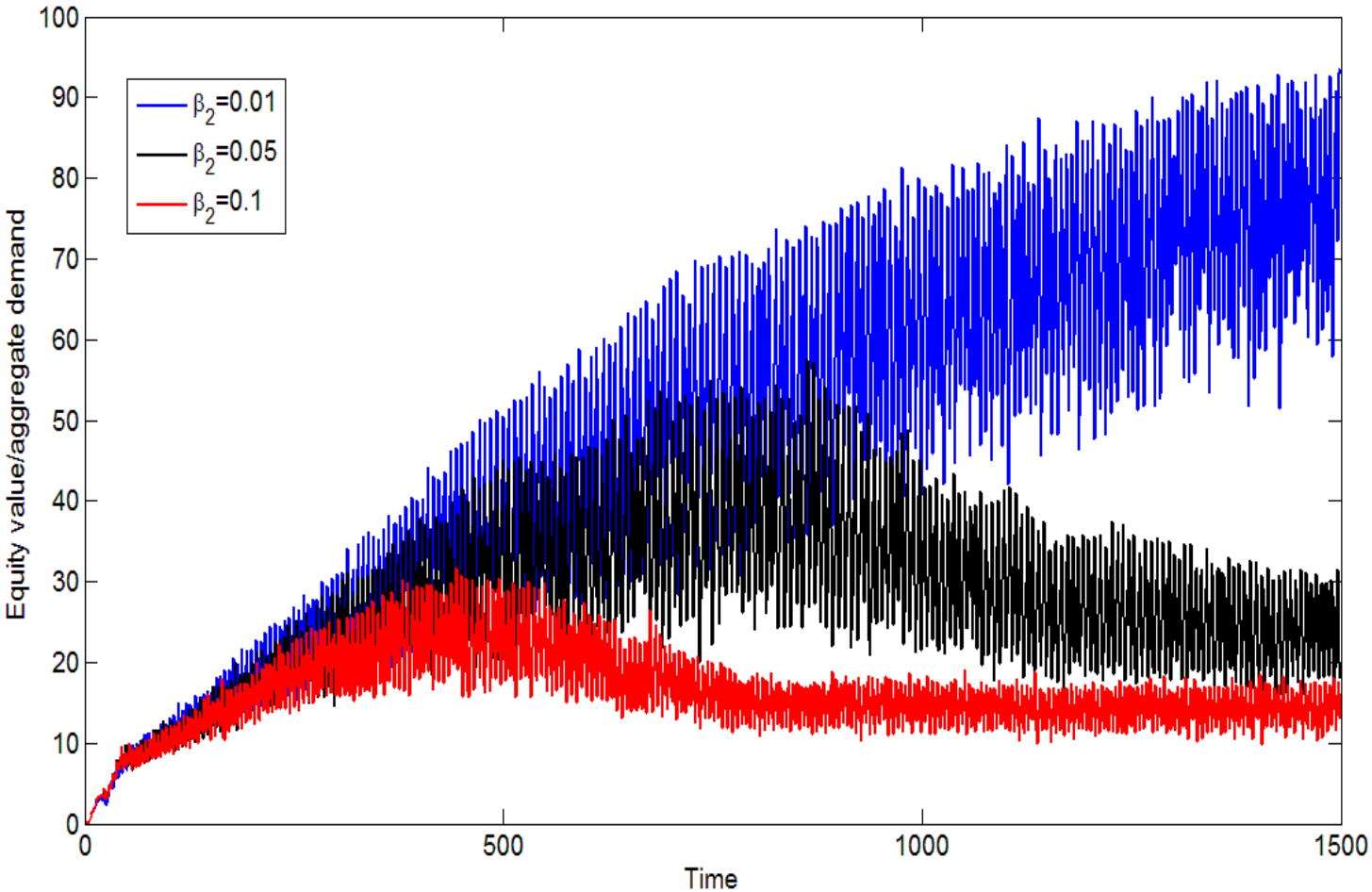


Figure: Dynamics of equity value to aggregate demand ratio for different values of β_2 ($\beta_1 = 0.05$).

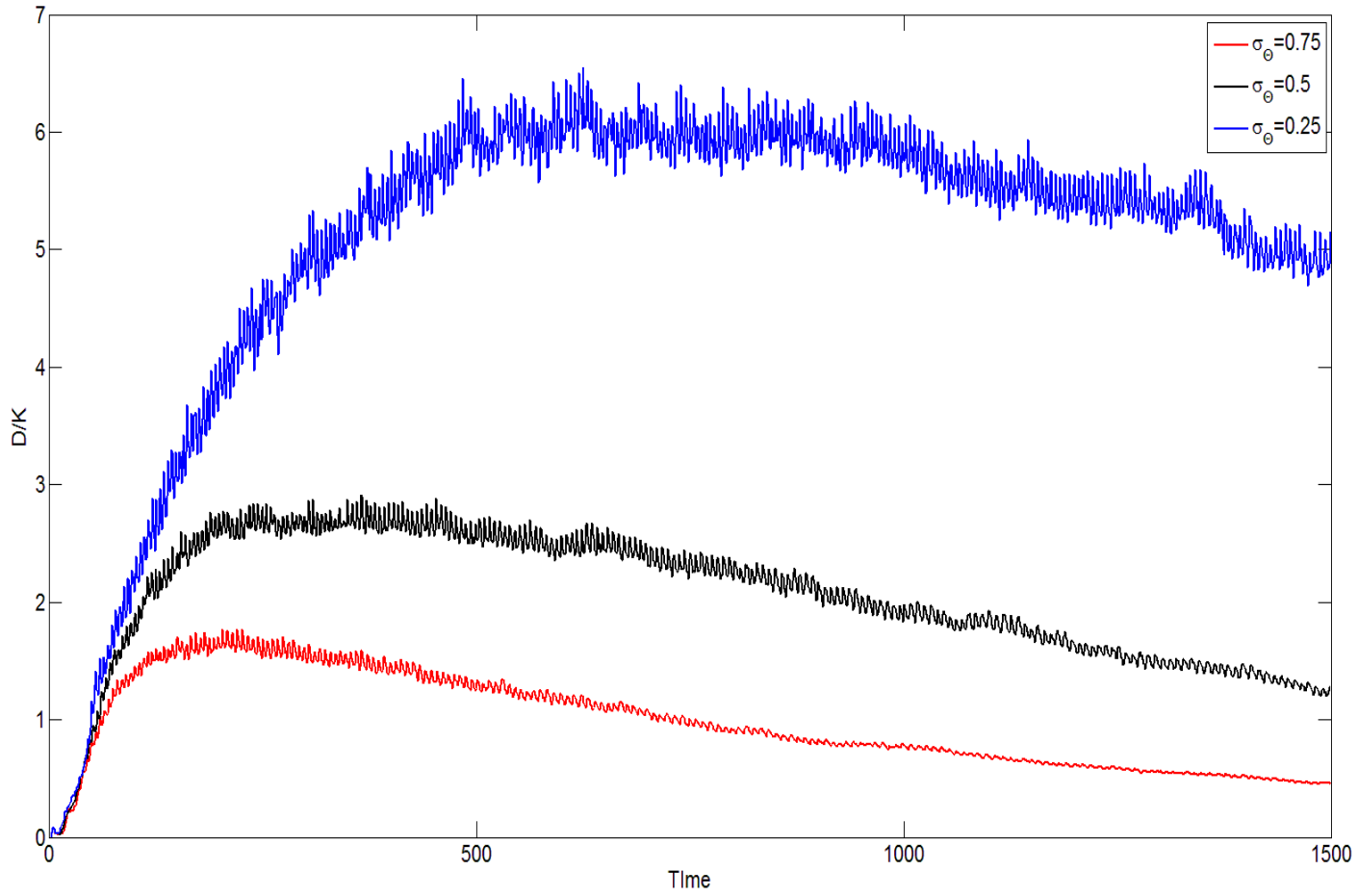


Figure: Dynamics of the debt/capital ratio for different values of σ_Θ.

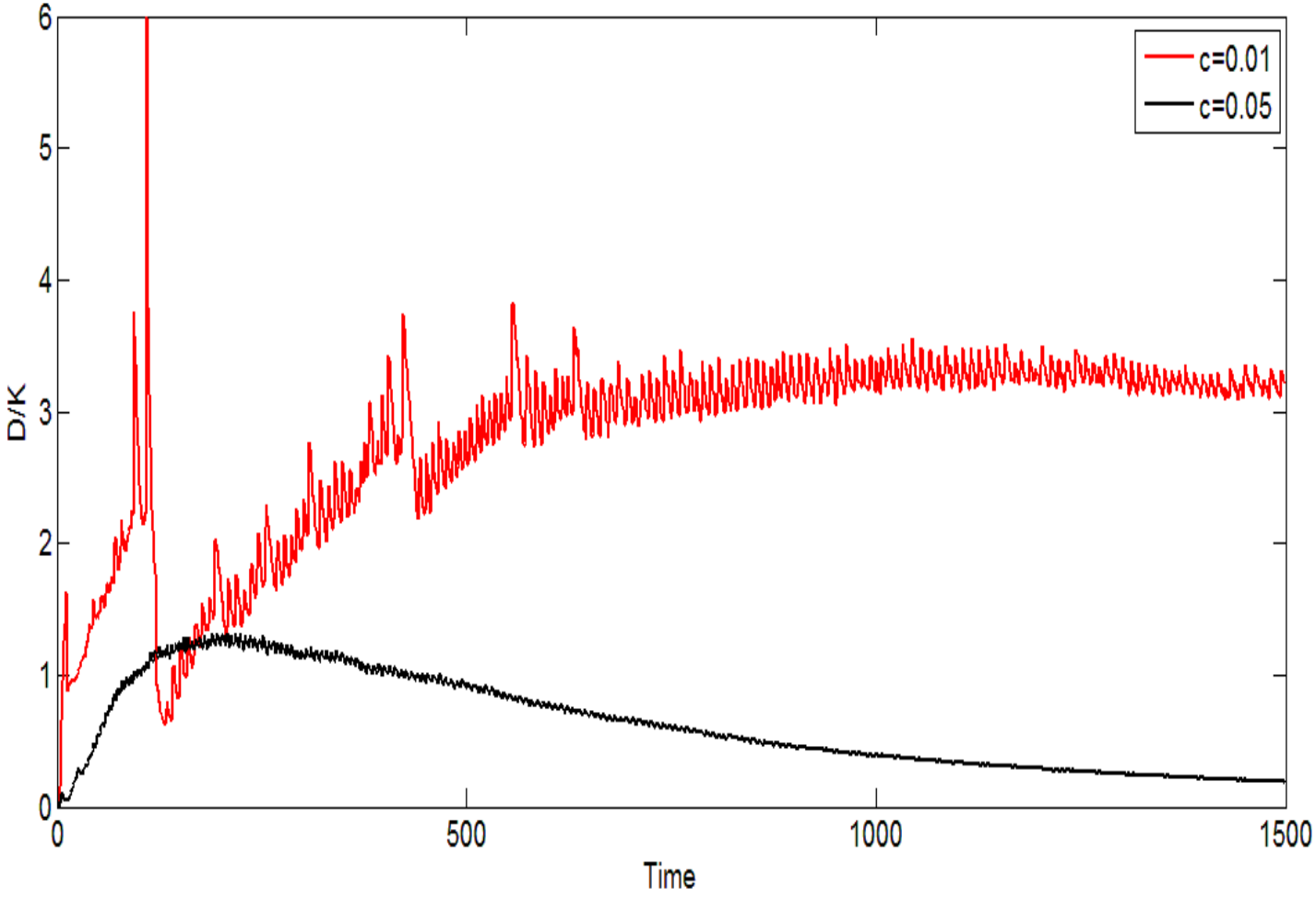


Figure: Dynamics of the debt/capital ratio for different values of c.

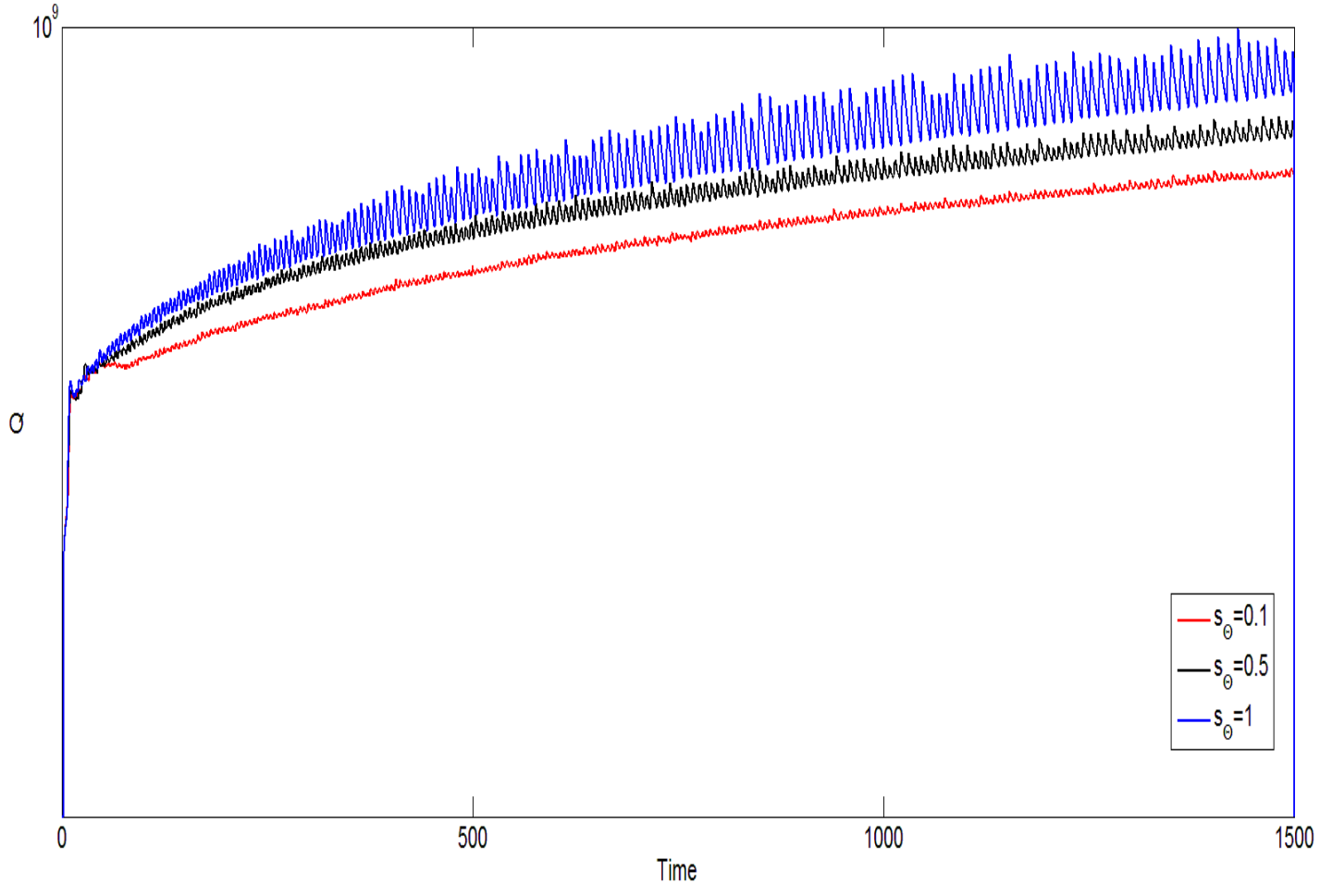


Figure: Dynamics of aggregate demand for different values of s_Θ.

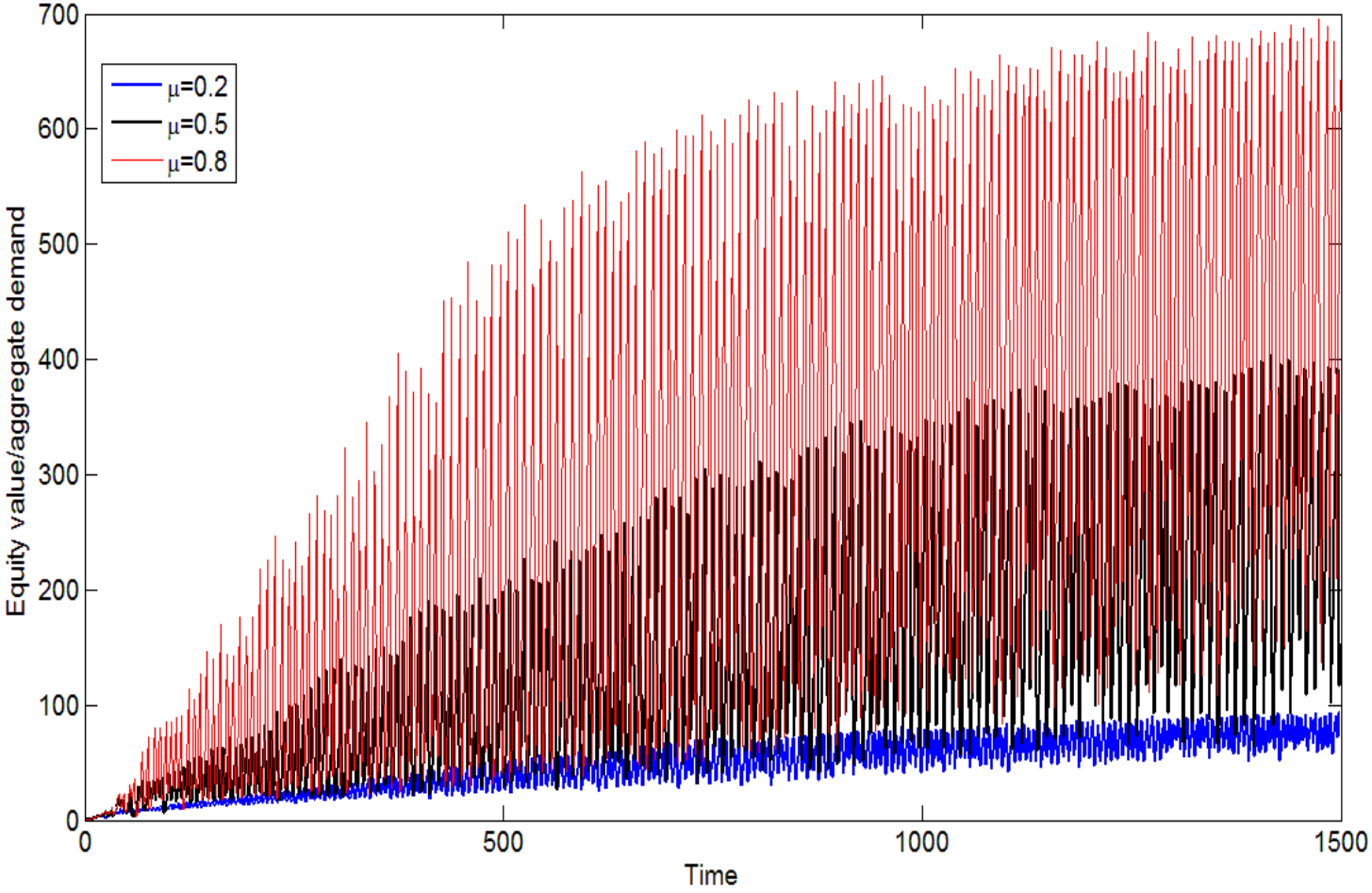


Figure: Dynamics of equity value to aggregate demand ratio for different values of μ.

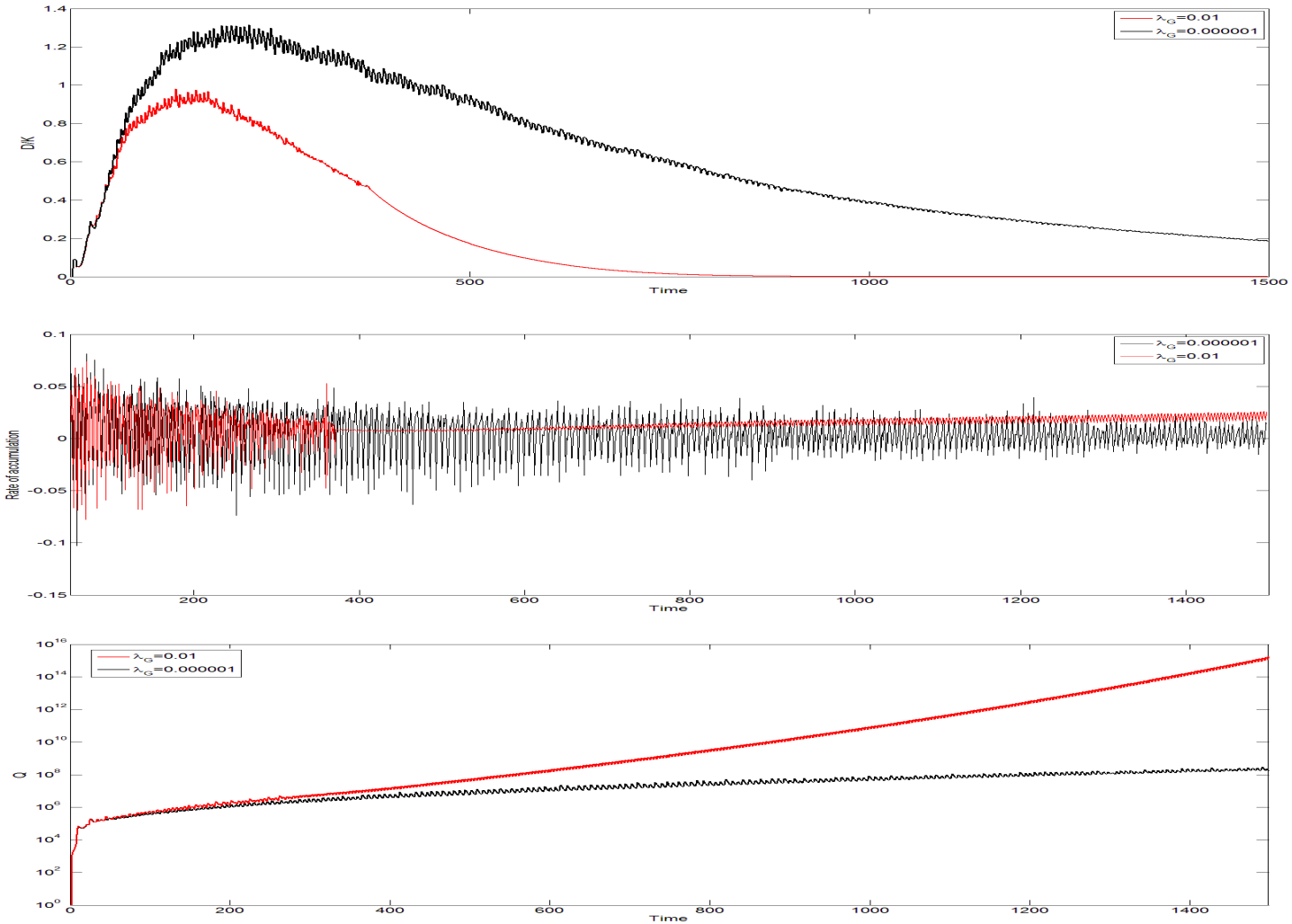


Figure: Debt to capital ratio (top), capital accumulation (center) and aggregate demand (bottom)for different values of λ_G.

- **Heterogeneity** in firms' behavior influences the dependence of the real sector to the financial sector (financialization and leverage ratio).
- the progressively larger weight of the **financial sector** can avoid the paradox of thrift and affects growth, amplitude of fluctuations and distribution of income.
- **inequality** (propensity to save and price mark-up) increases financialization and size of fluctuations;
- a lower level of the **preference for liquidity** causes higher demand and higher accumulation.

- A more refined study of the conditions under which bubbles and busts are generated in the present setting;
- introduction of a variable mark-up and the possibility for households to shift between the two categories of profit-earners and income-earners;
- a more sophisticated analysis of the analytical solution of the model, to be achieved by the study of the stability properties of the dynamical system.

Thank you!

- All firms adopt the same Leontief-type technology with constant coefficients. As a consequence, the demand for labor at full capacity can be residually quantified once the stock of capital is determined by investment decisions in the previous periods. The supply of labor is infinitely elastic.

$$\bar{q}^j(t) = 1/\gamma \; k^j(t) \tag{7}$$

$\gamma > 0$;

- Any excess of retained profits over investment will be held by the firm in the form of money m

$$\dot{m}^j(t) = a^j(t) - i^j(t) \tag{8}$$

- where savings S are defined as the difference between households' disposable income and consumption levels $S(t) = Y(t) - C(t)$, and $G(t') = [Pe(t') - Pe(t)]E(t)$.
- The demand for money is residually determined as

$$M_h(t) = W(t) - Pe(t)E(t) \tag{9}$$

- Given that only profit earners demand for share, we have that

$$\dot{M}_\Psi(t) = Y_\Psi(t) - C_\Psi(t) \tag{10}$$

and, accordingly

$$M_\Theta(t) = M_h(t) - M_\Psi(t) \tag{11}$$

- Total output Q is divided between aggregate consumption C and investment I:

$$pQ(t) = (1-s_\Psi)Y_\Psi(t)+(1-s_\Theta)Y_\Theta(t)+(1-\sigma_\Theta)G_\Theta(t)+I(t)$$

- Accordingly

$$pQ(t) = \frac{1+\mu}{s_\Psi - \mu[1-\Theta(1-s_\Theta)]}[I(t)+A(t)] \qquad (12)$$

where

$$A(t) = r[(1-s_\Psi)M_\Psi(t)+(1-s_\Theta)(M_\Theta(t)-\Theta B(t))]+(1-\sigma_\Theta)G_\Theta(t)$$

- Probability for a firm of transitioning from one state to another:

$$\eta^j(t) = Pr[s(t) \geq \Gamma_1], \tag{13}$$

$$\zeta^j(t) = Pr[s(t) < \Gamma_2]. \tag{14}$$

where

$$\Gamma_z = \frac{i_z(t) - m_z(t) - [\Pi p q_z(t) - r(b_z(t) - m_z(t)]}{(1-\Theta)\Pi p q_z(t)} \frac{K(t)}{K(t) - K_z(t)}. \tag{15}$$

$$\begin{aligned} \frac{dP(N_1,t)}{dt} = \quad & \omega_+(t)P(N_1-1)(t) + \omega_-(t)P(N_1+1)(t) \\ & + [\omega_+(t) + \omega_-(t)]\, P(N_1)(t) \end{aligned} \tag{16}$$

- The solution method splits the state variable into 2 components

$$N_1 = Nm + \sqrt{N}s \tag{17}$$

m is the deterministic trend; s is the stochastic noise

- trend dynamics:

$$\frac{dm}{d\tau} = \eta m - (\eta + \zeta)m^2, \tag{18}$$

where $\tau = t/N$.

- stationary distribution of the Fokker-Planck equation:

$$\theta(s) = C \exp\left(-\frac{s^2}{2\sigma^2}\right) \quad : \quad \sigma^2 = \frac{\eta\zeta}{(\eta+\zeta)^2}. \tag{19}$$

- dynamics of n_1:

$$\frac{n_1(t)}{dt} = \eta m - (\eta + \zeta)m^2 + \sigma\, dV(t) \tag{20}$$

dV is a stationary Wiener increment.

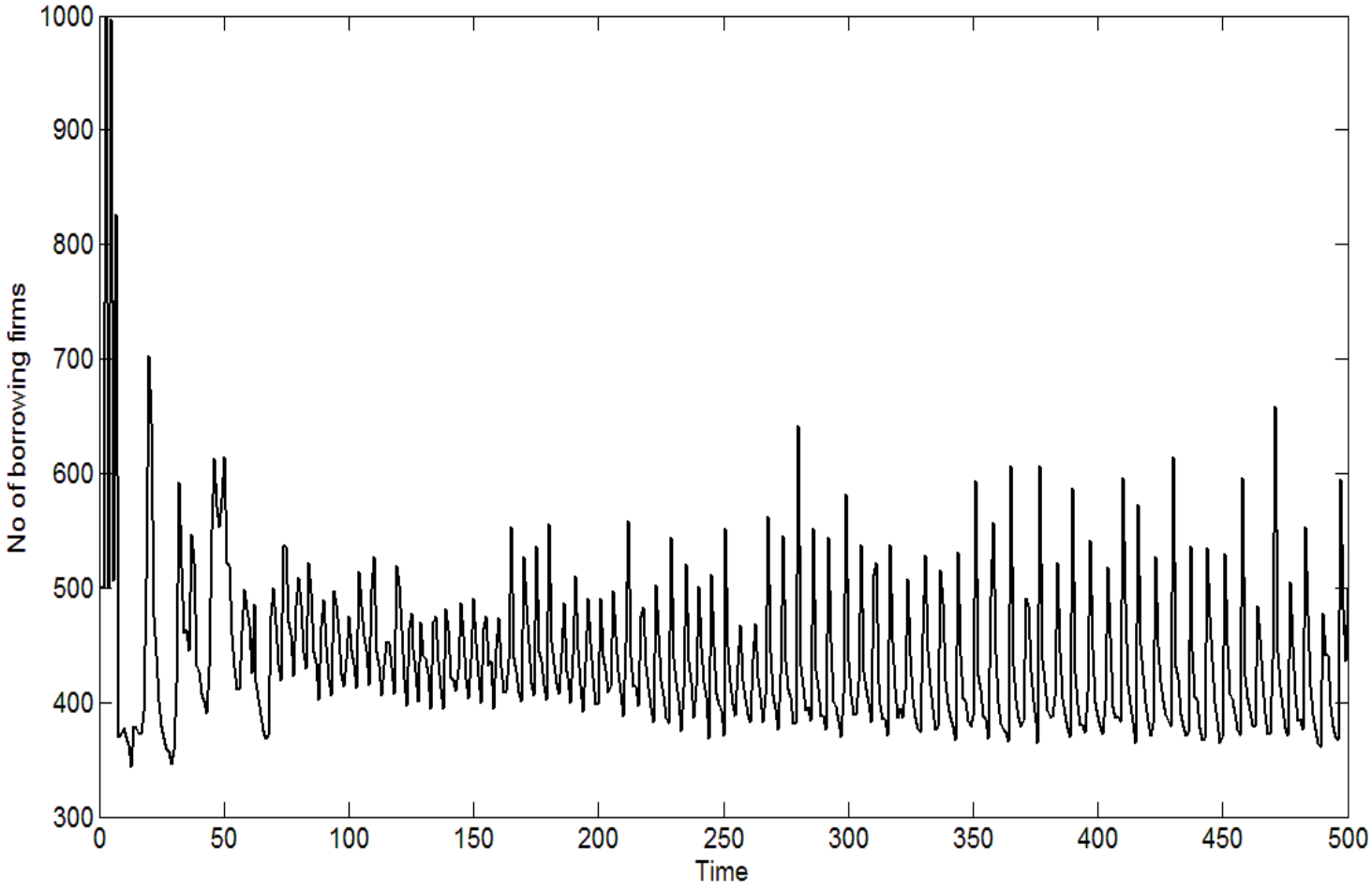


Figure: Dynamics of the number of borrowing firms (total number of firms: 1,000).

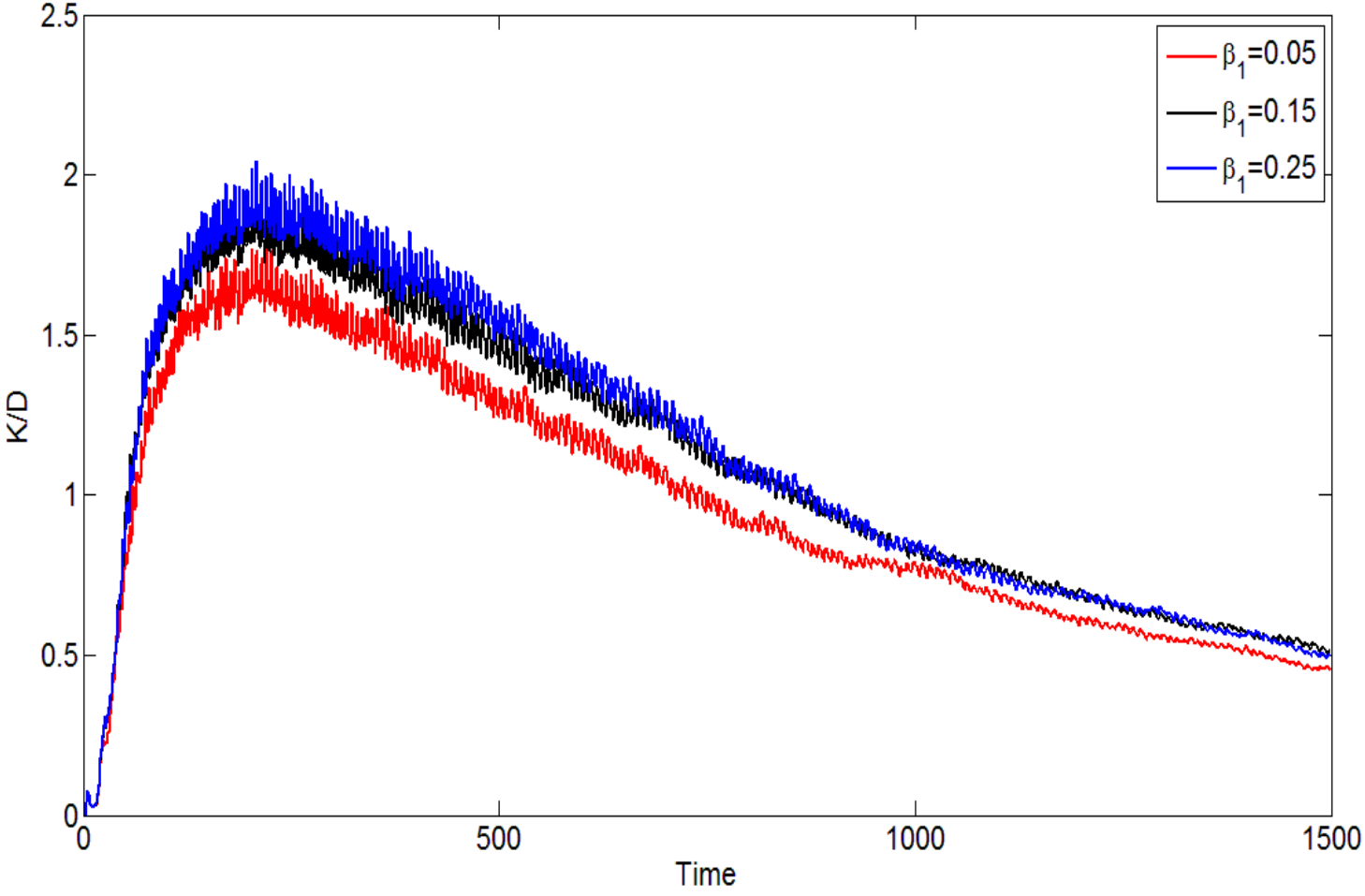


Figure: Dynamics of the debt/capital ratio for different values of β_1.

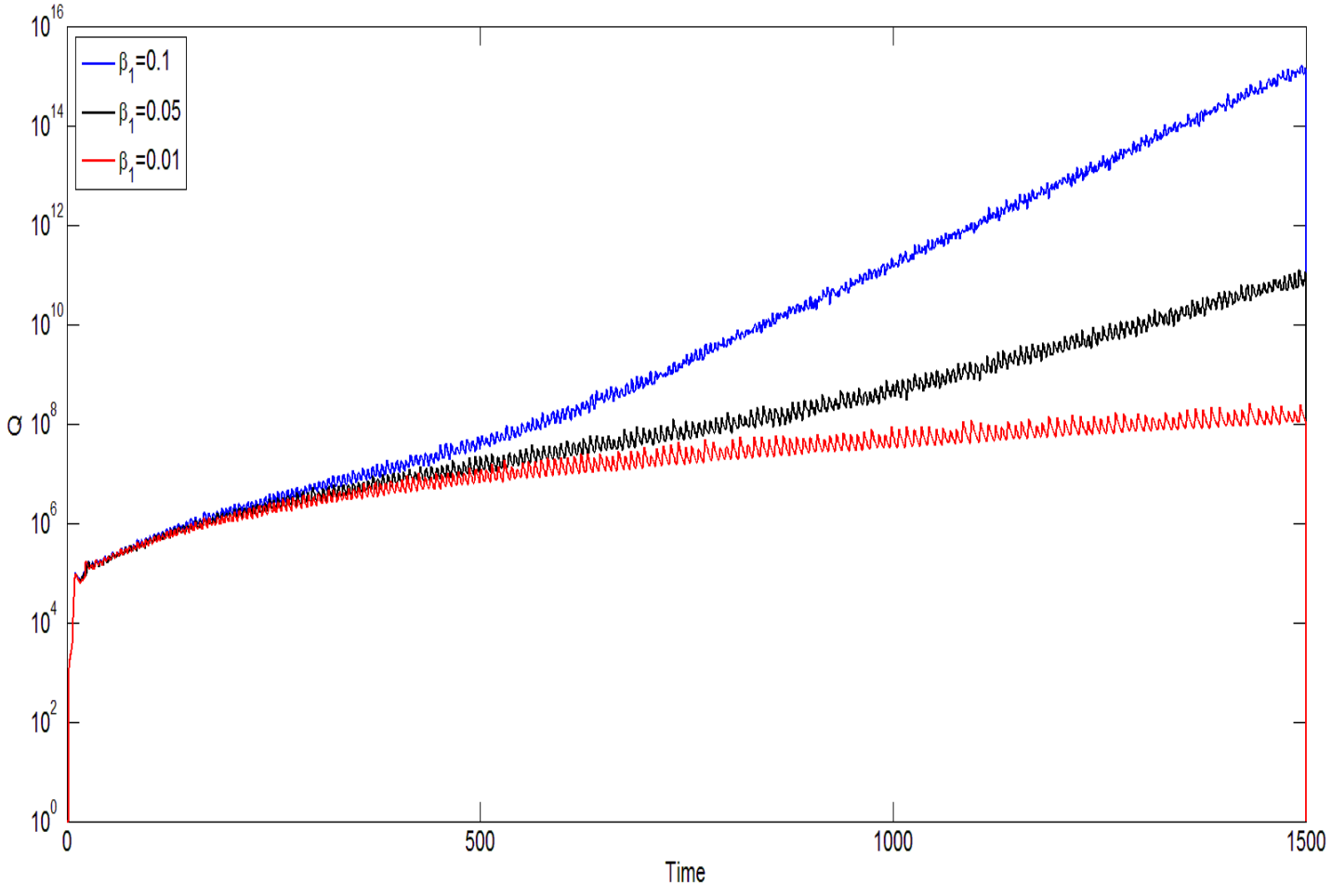


Figure: Dynamics of aggregate demand for different values of β_2.

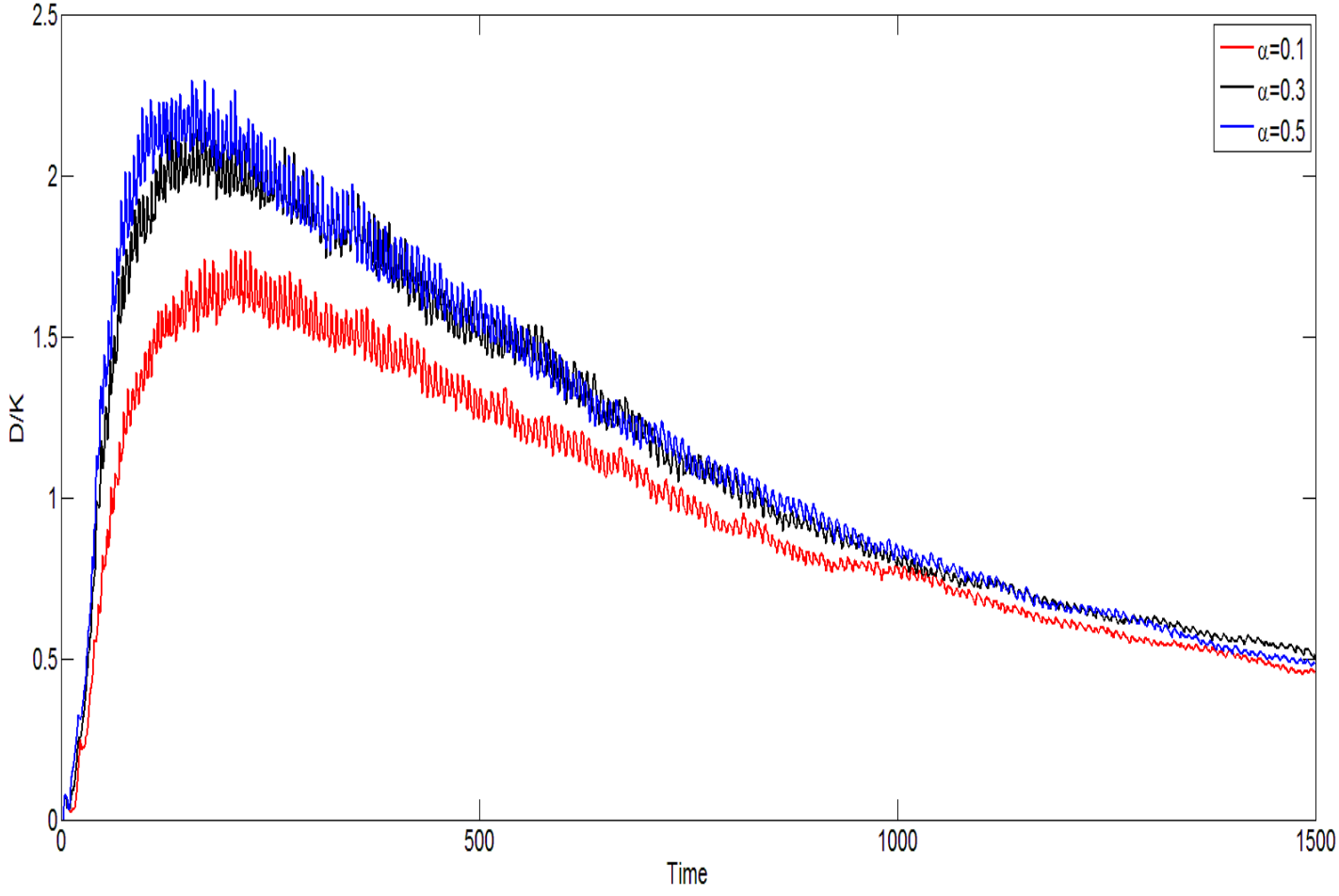


Figure: Dynamics of the debt/capital ratio for different values of α.

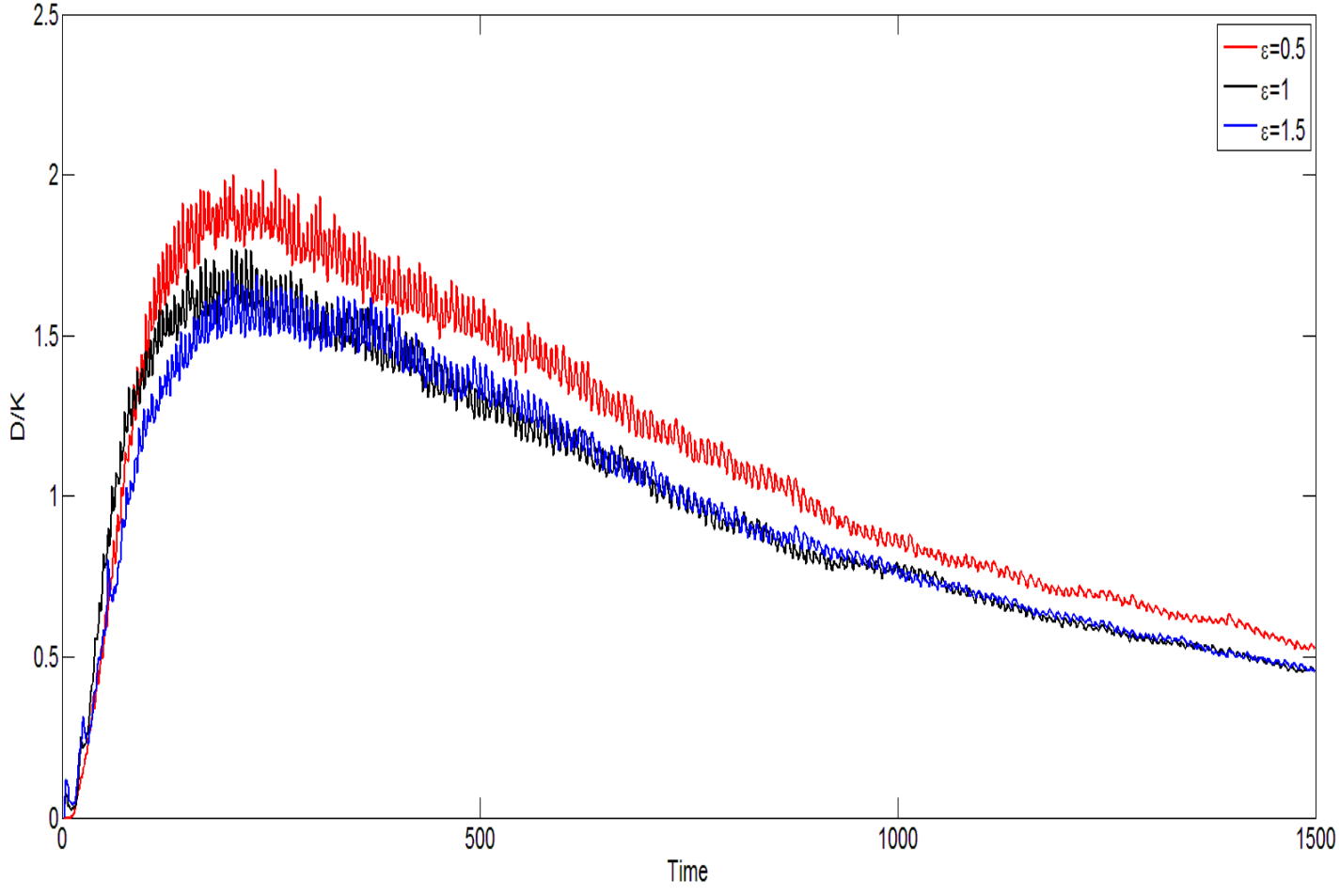


Figure: Dynamics of the debt/capital ratio for different values of ϵ.

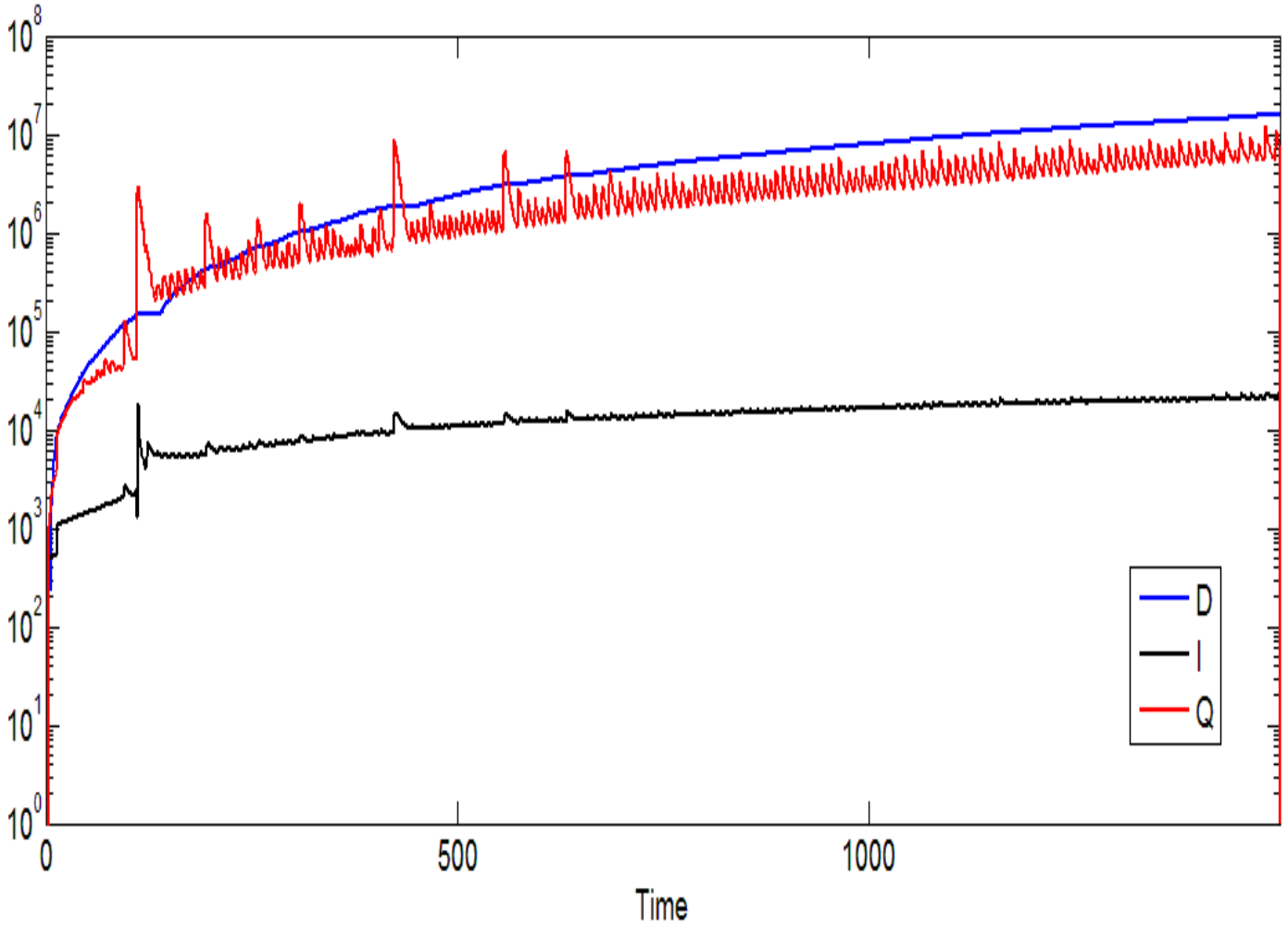
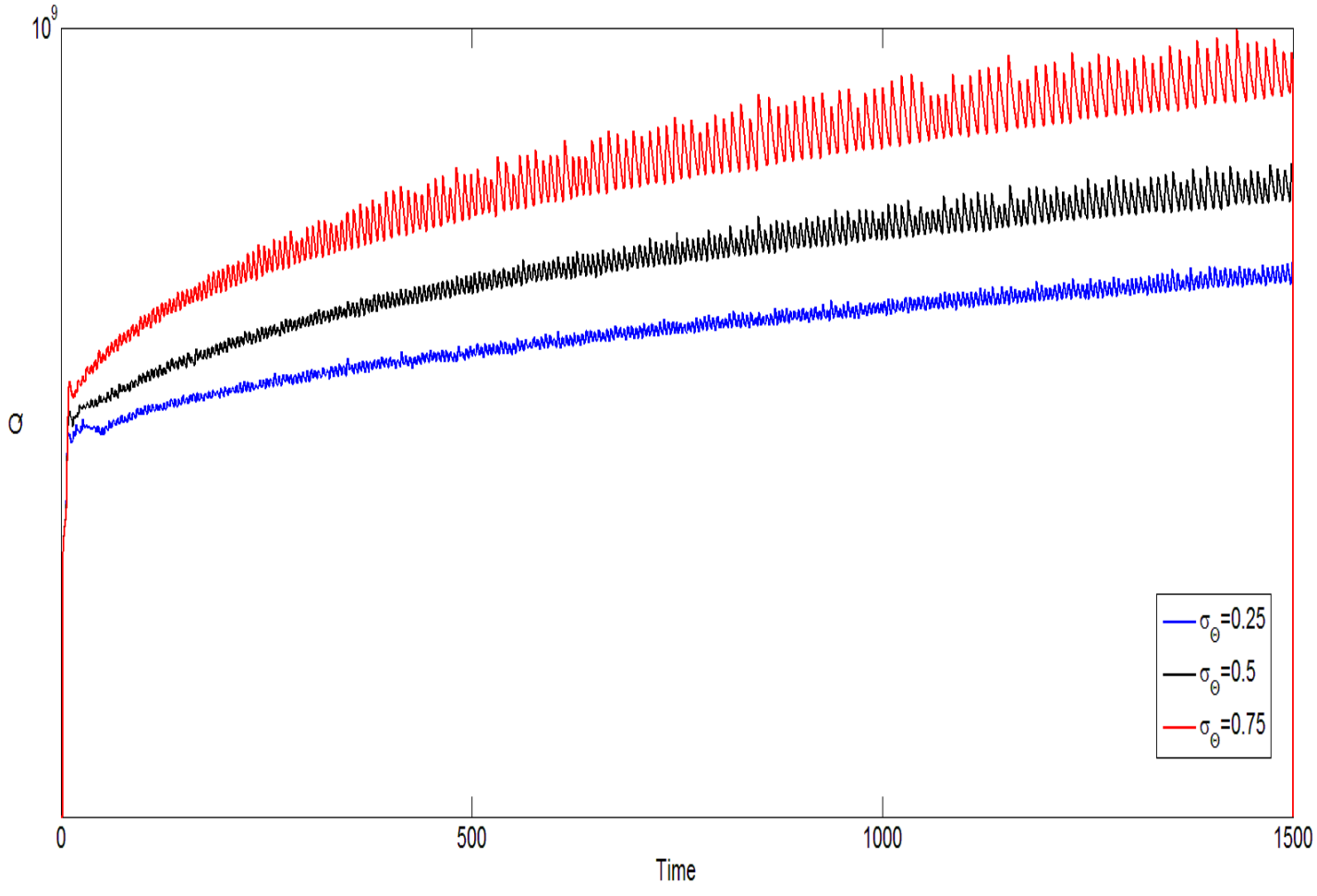


Figure: Dynamics of debt, investment and aggregate demand for $c = 0.01$.



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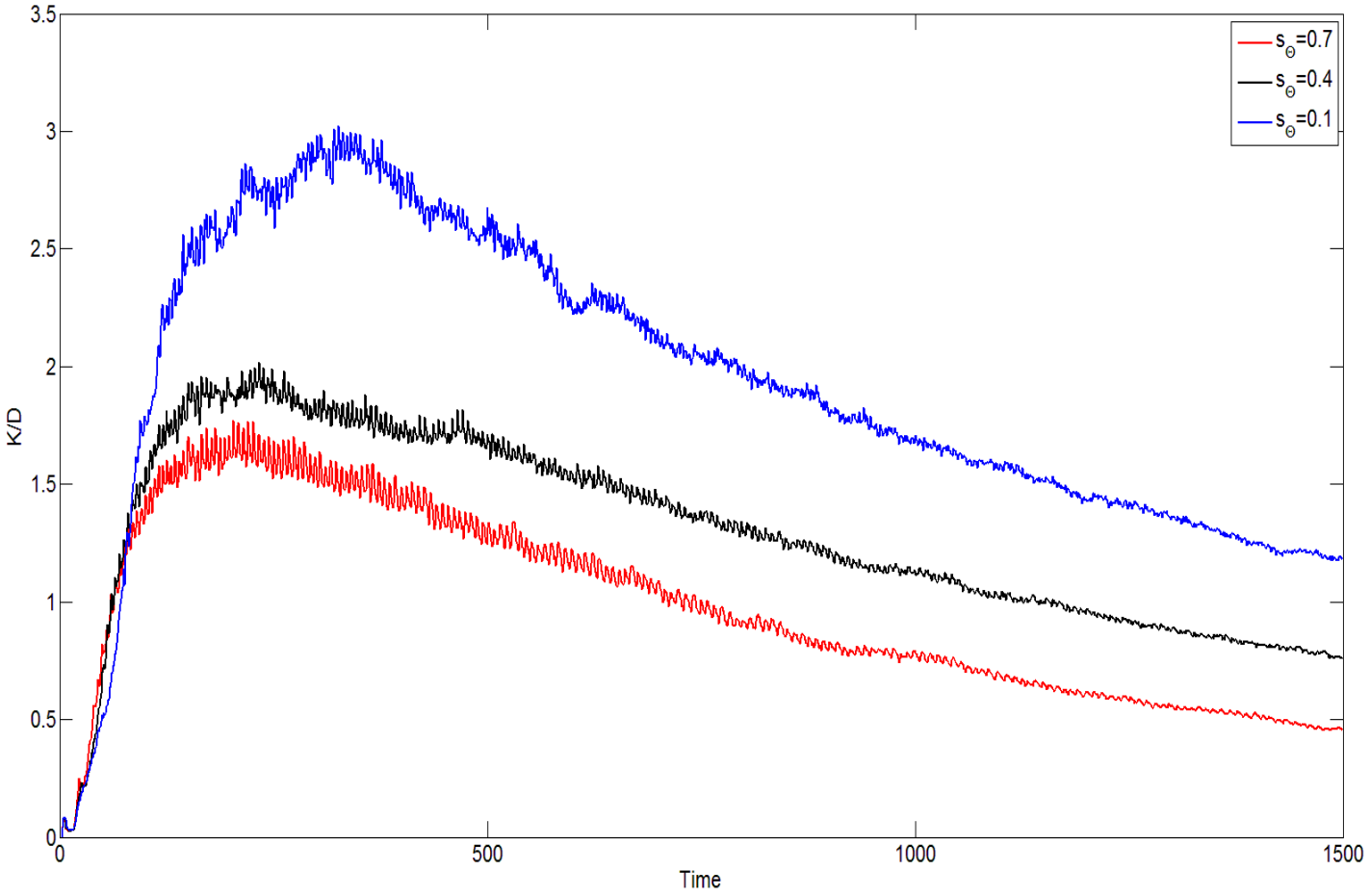

Figure: Dynamics of the debt/capital ratio for different values of s_Θ.