Macroeconomic Modelling for Multi-Agent Economies with Financing Constraint

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From E. Leamer, Macroeconomic Patterns and Stories

- "Ideally what we (macroeconomists) should be seeking are clear patterns and corresponding compelling stories.
- Most of what economists believe comes from the stories that we tell each other. Economists are story telling animals
- Pictures, Words and Numbers
  - pictures referring to the patterns
  - words to the stories
  - numbers to the scientific analysis"

Here is an interesting pattern

- When technology is linear, x changes one-to-one with the output gap

Can we replicate this pattern? Sure...

- A linear macroeconomic model (with or without microfoundations) can do it, provided you pepper the model with systematic random shocks characterized by an appropriate autoregressive structure
- A non-linear macroeconomic model can also fit the pattern, without random shocks but with an appropriate dynamic deterministic skeleton (and a very specific calibration)
- Last but not least, a macroeconomic agent based model (**MABM**) can do an excellent job in replicating the pattern

A macroeconomic pattern is an emergent property...

- In a MABM aggregate variables such as GDP, consumption etc. are computed *"from the bottom up"*, i.e. summing individual quantities across agents
- The time series of artificial macroeconomic data generated by the MABM through simulations follow the same pattern of *irregular fluctuations* (around a quasi-stationary long run "equilibrium") shown by the empirical time series of GDP or the output gap
- This pattern is an *emergent property* of the macroeconomic system due to the interaction of heterogenous agents
- Usually we MABM fans are blissfully happy with this characterization...

... but this is not enough

- When a change of an exogenous variable (may be a policy move) generates changes in the macroeconomic pattern generated by the model we are (almost) in the dark ...*interpretation is difficult and somehow arbitrary*
- This is the standard criticism: ABMs are *"black boxes"*
- In Leamer's wording we don't have *compelling stories* to tell about the transmission of shocks in a MABM

A hybrid Macroeconomic and AB Model (M&ABM)

- In the paper *E Pluribus Unum* , joint with Tiziana Assenza we suggest a way to *reconcile macroeconomic thinking and AB modelling*
- We nest an ABM into a standard macro model therefore building a hybrid macroeconomic and agent based model **(M&ABM)**
- *We want to gain clarity* in the conclusions we can get from M&ABM. In order to reach this goal we sacrifice – at least in part – the richness of the macroeconomic representation of MABM

Plan of the talk

- M&ABM: The Macroeconomic Model
- M&ABM: The ABM
- The effects of a financial shock
- Conclusions

The environment

- Closed economy: households, firms, the public sector (a Government and a central bank), financial intermediaries.
- Firms are heterogeneous in terms of financial robustness; households are homogeneous.
- There are markets for goods, money, bonds, credit.
- Wages and prices are constant

Fundamental assumptions

- Firms differ according to the *equity ratio* (the ratio of the individual equity base or net worth to the individual capital stock)
  $a_{it} = \frac{A_{it}}{K_{it}}; i = 1, 2, .., F.$
- Probability of bankruptcy ($\Phi_{it} = \frac{\alpha}{a_{it-1}}$) decreasing with the equity ratio: the higher the equity ratio, the lower the probability of bankruptcy
- The firm goes bankrupt if the equity ratio falls below α *(bankruptcy threshold).*

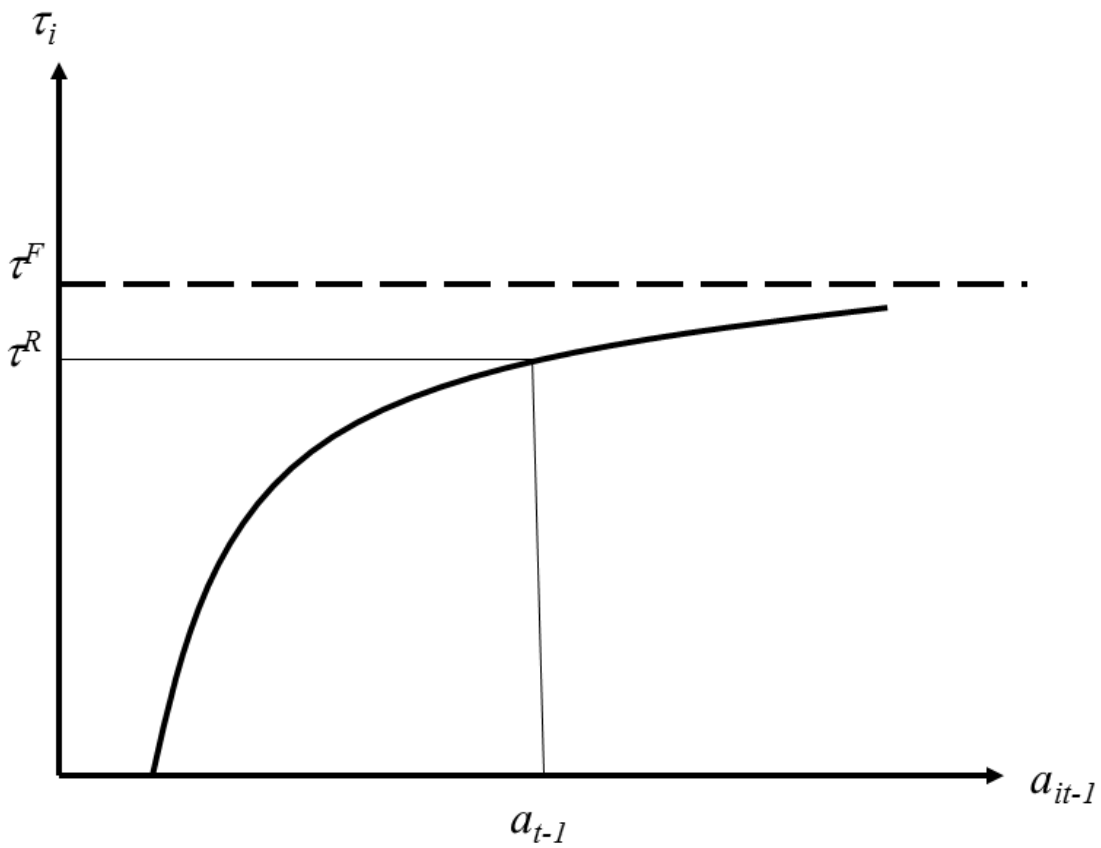
The problem of the firm

- Each firm maximizes expected profit less bankruptcy costs times the probability of bankruptcy Φ_{it} (Greenwald and Stiglitz, 1993).
- From the FOC one gets the individual *investment ratio* (IR):

$$\tau_{it} = \gamma - (r_t + f_{it}) \tag{1}$$

- $\tau_{it} \equiv \frac{I_{it}}{\bar{K}_{t-1}}$ with $\bar{K}_{t-1}$ the average capital stock
- r_t is the interest rate (uniform across firms).
- $f_{it} = \beta\Phi_{it} = \frac{\beta\alpha}{a_{it-1}}$ is the *external finance premium* (EFP)

Investment ratio and equity ratio



The average investment ratio

- In order to aggregate individual behaviour, we take an approximation of the individual IR up to the second term in the neighborhood of the average equity ratio $< a_{it-1} >= a_{t-1}$

$$\tau_{it} \approx \tau_t^R + \frac{\partial \tau_{it}}{\partial a_{it-1}} \left(a_{it-1} - a_{t-1}\right) + \frac{1}{2} \frac{\partial^2 \tau_{it}}{\partial a_{it-1}^2} \left(a_{it-1} - a_{t-1}\right)^2$$

where τ_t^R is the IR of the Representative Agent

The average investment ratio (cont'd)

- The *average* investment ratio is:

$$\tau_t = < \tau_{it} > = \tau_t^R + \frac{1}{2} \frac{\partial^2 \tau_{it}}{\partial a_{it-1}^2} \underbrace{< (a_{it-1} - a_{t-1})^2 >}_{V_{t-1}}$$

i.e.

$$\tau_t = \gamma - [r_t + f(a_{t-1}, V_{t-1})] \tag{2}$$

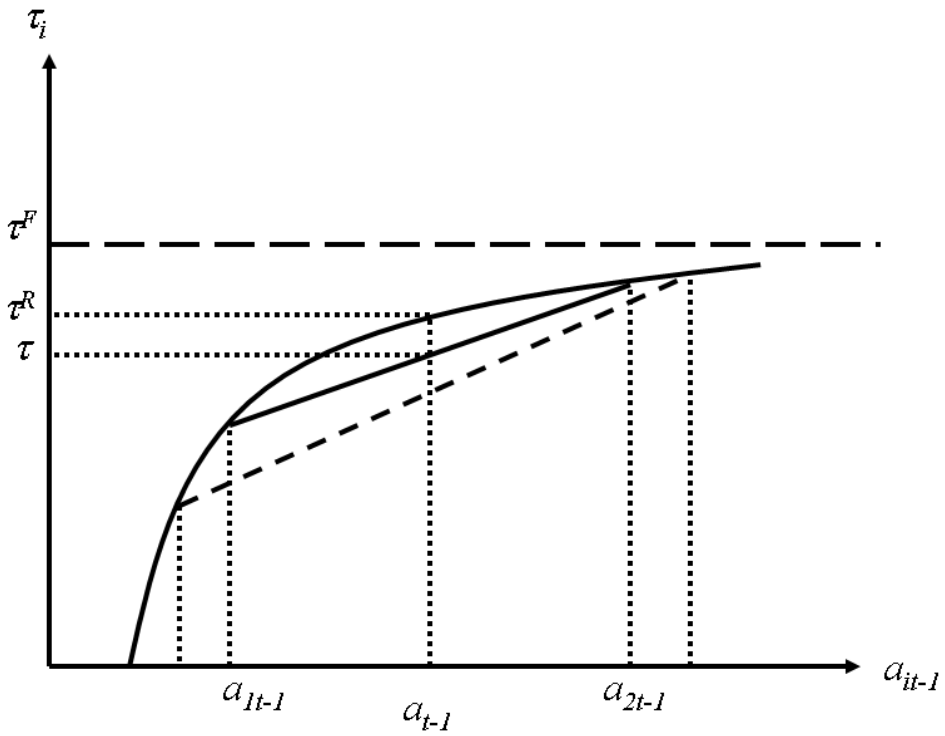
where

$$f(a_{t-1}, V_{t-1}) := \frac{\beta\alpha}{a_{t-1}} \left(1 + \frac{V_{t-1}}{a_{t-1}^2}\right)$$

The average investment ratio (cont'd)

- The average IR τ_t is affected by the average (economywide) EFP $f\left(a_{t-1}, V_{t-1}\right)$ which takes into account the shape of the distribution of the equity ratio captured by the moments.

The effect of an increase in the variance of the equity ratio on the average investment ratio



Assumptions

- For the sake of simplicity, we assume that there is a representative household
- Each member of the household may be employed with probability x_t
- $1 - x_t$ coincides with the fraction of household's members who are unemployed (unemployment rate)
- The member earns the wage rate w if employed and the unemployment subsidy σ if unemployed, $w > \sigma$
- Therefore the household's income is $wx_t + \sigma(1 - x_t) = \sigma + \omega x_t$ with $\omega := w - \sigma$
- Notice that x_t can be interpreted as a measure of capacity utilization or of the *output gap*

Consumption and the demand for money

- From the *FOCs* for utility maximization we get

$$\begin{cases} c_t = \sigma + \omega x_t \\ m_t^d = \dfrac{1 + r_t}{r_t} (\sigma + \omega x_t) \end{cases}$$

- Optimal consumption and money demand are functions of the output gap x_t and the interest rate r_t

The augmented IS curve

- Equilibrium on the goods market (C+I=Y), after some algebra yields:

$$r_t = \zeta (1 - x_t) - f (a_{t-1}, V_{t-1}) \tag{3}$$

- This is the (optimizing) *IS curve* on the (r_t, x_t) plane
- *The moments of the distribution of the equity ratio are shift parameters of the IS curve*

LM curve

- The equilibrium condition on the money market $\overline{M}_t^s = M_t^d$ yields the *LM curve*

$$x_t = \frac{1}{\omega}\left(r_t \bar{m}_t^s - \sigma\right) \tag{4}$$

where $\bar{m}_t^s$ is per-capita money supply.

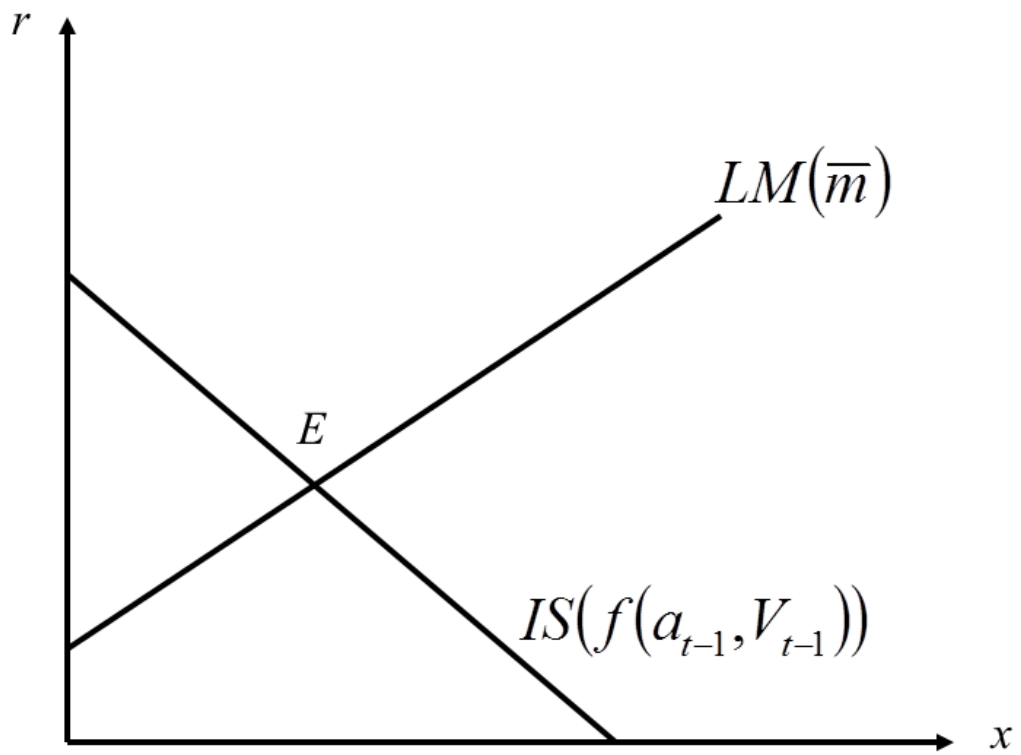
The reduced form

- The reduced form of (3) (4) is:

$$\begin{cases} r_t = \Gamma_0 \left[\zeta w - \omega f\left(a_{t-1}, V_{t-1}\right)\right] \\ x_t = \frac{1}{\omega}\left\{\bar{m}\Gamma_0 \left[\zeta w - \omega f\left(a_{t-1}, V_{t-1}\right)\right] - \sigma\right\} \end{cases}$$

  where $\Gamma_0 = (\omega + \zeta\bar{m})^{-1}$ is positive and decreasing with money supply
- All the endogenous variables in equilibrium turn out to be functions of the moments of the distribution of the equity ratio

The "moments augmented" IS-LM model



The Law of Motion of the Equity Ratio

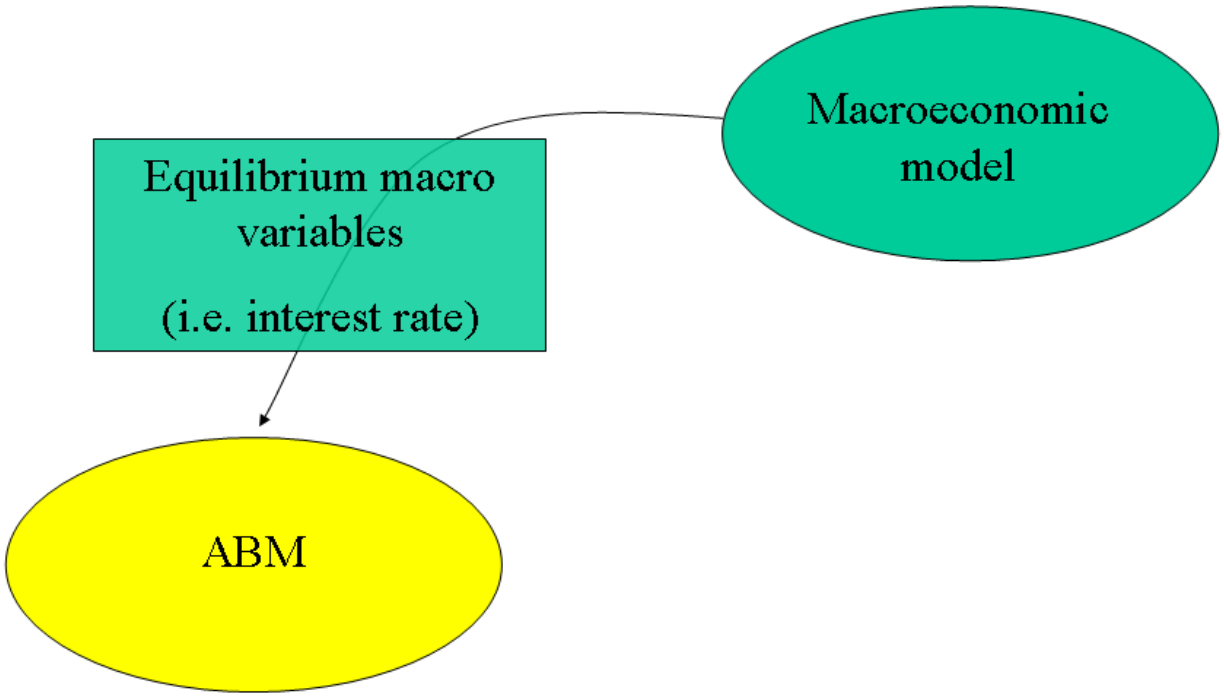
- The moments of the distribution change over time because *the equity ratio at the micro level* follows a specific law of motion
- To determine this law, start from the level of net worth $A_{it} = A_{it-1} + \pi_{it}$
- Dividing by K_{it} we get

$$a_{it} = a_{it-1}\left(\frac{1}{1+\tau_{it}}\right) + u_{it}\nu - \frac{w}{\sigma}\zeta - r_t - \frac{1}{2}\frac{\tau_{it}^2}{1+\tau_{it}}$$

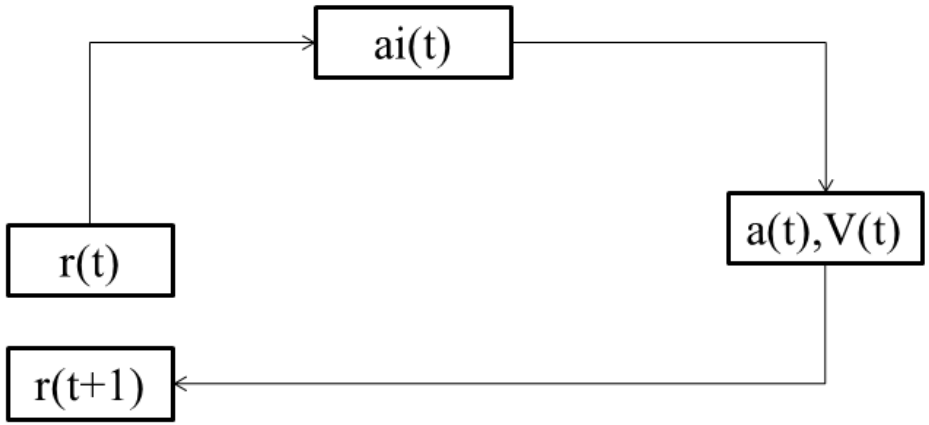
The Law of Motion of the Equity Ratio (cont'd)

- u_i is random shock with $E(u_i) = 1$
- $\tau_{it} = \gamma - (r_t + f_{it})$ is the individual investment ratio, where $f_{it} = \dfrac{\beta\alpha}{a_{it-1}}$ is the EFP
- The interest rate r_t is determined *at the macro level in equilibrium* as shown above: $r_t = \Gamma_0 \left[\zeta w - \omega f\left(a_{t-1}, V_{t-1}\right)\right]$

Feedback 1: from macro to ABM



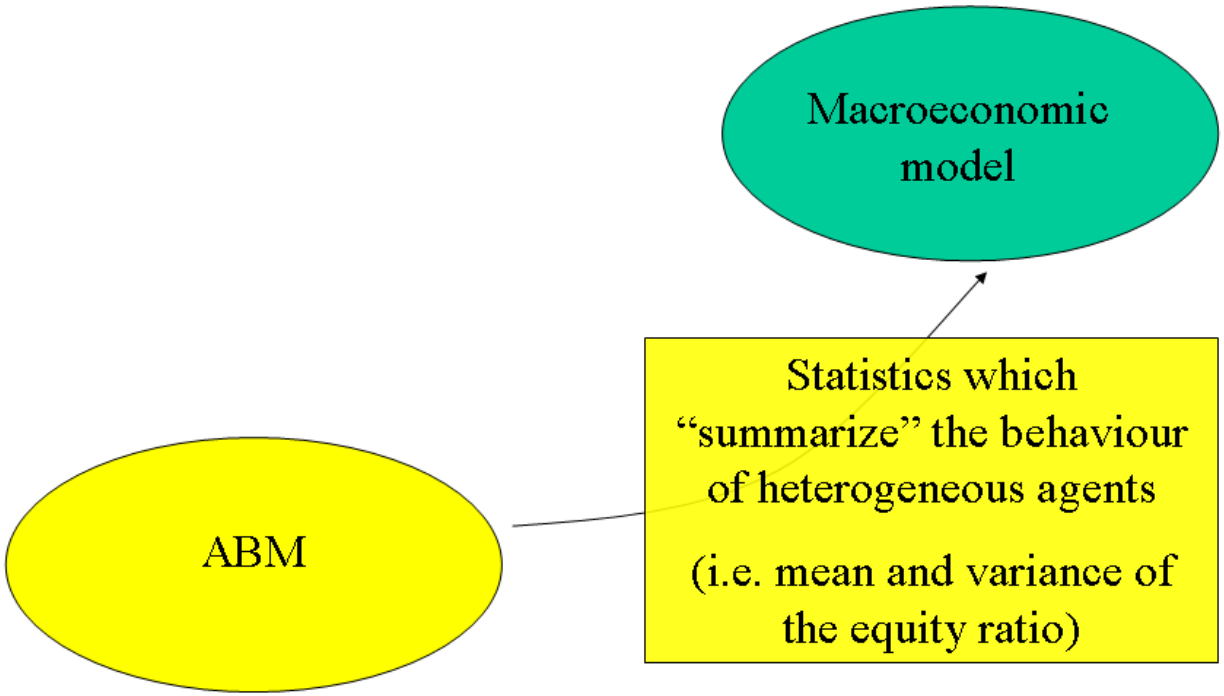
Macroeconomic externality



Simulation procedure

- Virtual economy consisting of $F = 1000$ firms over a time span of $T = 1000$ (quarters)
- Only 7 free parameters
- From artificial data can obtain the time series of the cross sectional mean and variance $a_t, V_t; t = 1, 2, ..., T$

Feedback 2: from ABM to macro



The search for the moments

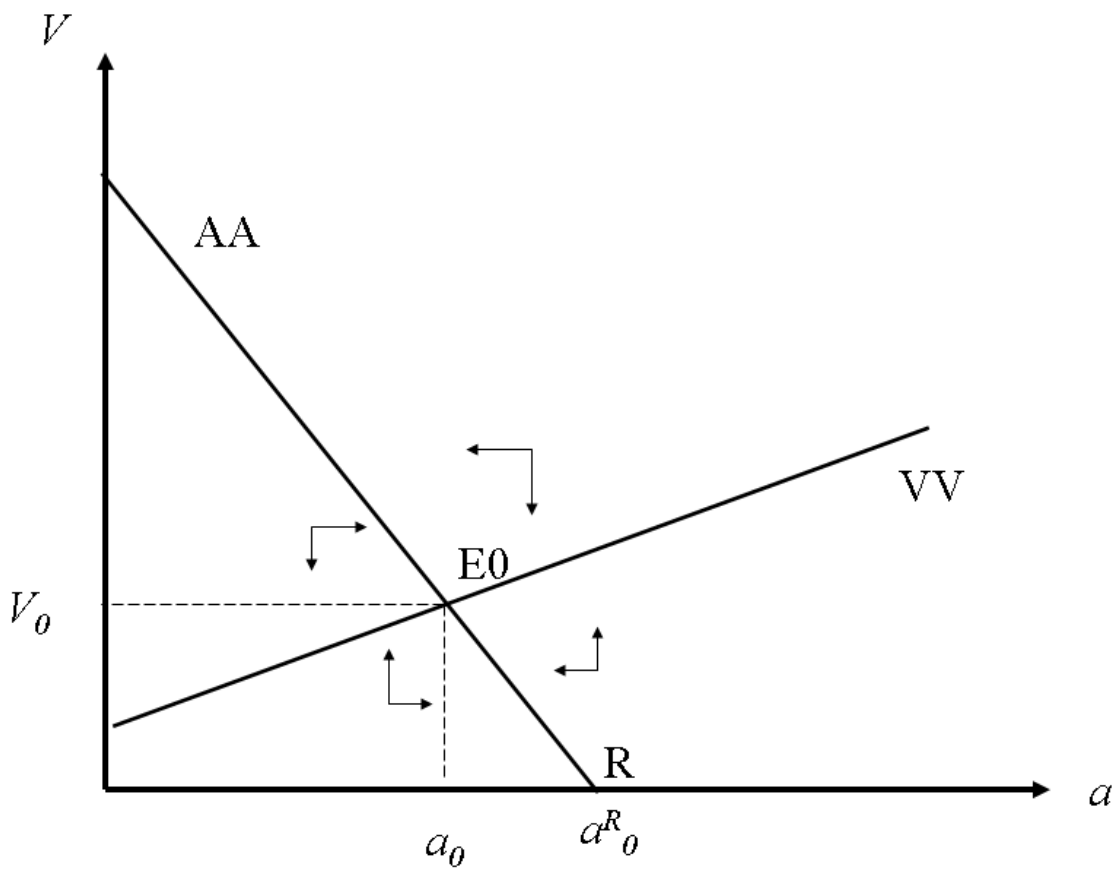
- In order to close the macroeconomic model we need to know how the moments of the distribution evolve in time
- We assume that the "true" law of motion of the cross sectional mean and variance may be approximated by means of a linear bivariate $AR(1)$ process
- We run an OLS regression on the time series of the cross-sectional mean and variance a_t and V_t to estimate the coefficients of the following linear system

$$\begin{aligned} a_t &= \alpha_{10} + \alpha_{11} a_{t-1} + \alpha_{12} V_{t-1} \\ V_t &= \alpha_{20} + \alpha_{21} a_{t-1} + \alpha_{22} V_{t-1} \end{aligned}$$

The distribution of the equity ratio in steady state

- The estimated α coefficients are all significant. In our simulations α_{12} and α_{21} are negative
- We can visualize the system using demarcation curves on the (a_t, V_t) plane
  - The condition $\Delta a = 0$ yields the downward sloping AA curve
  - The condition $\Delta V = 0$ yields the upward sloping VV curve
- The steady state of this system are the moments $E_0 = (a_0, V_0)$ of the *long run distribution* of the equity ratio
- This steady state is stable

The phase diagram



The EFP, interest rate and output gap in steady state

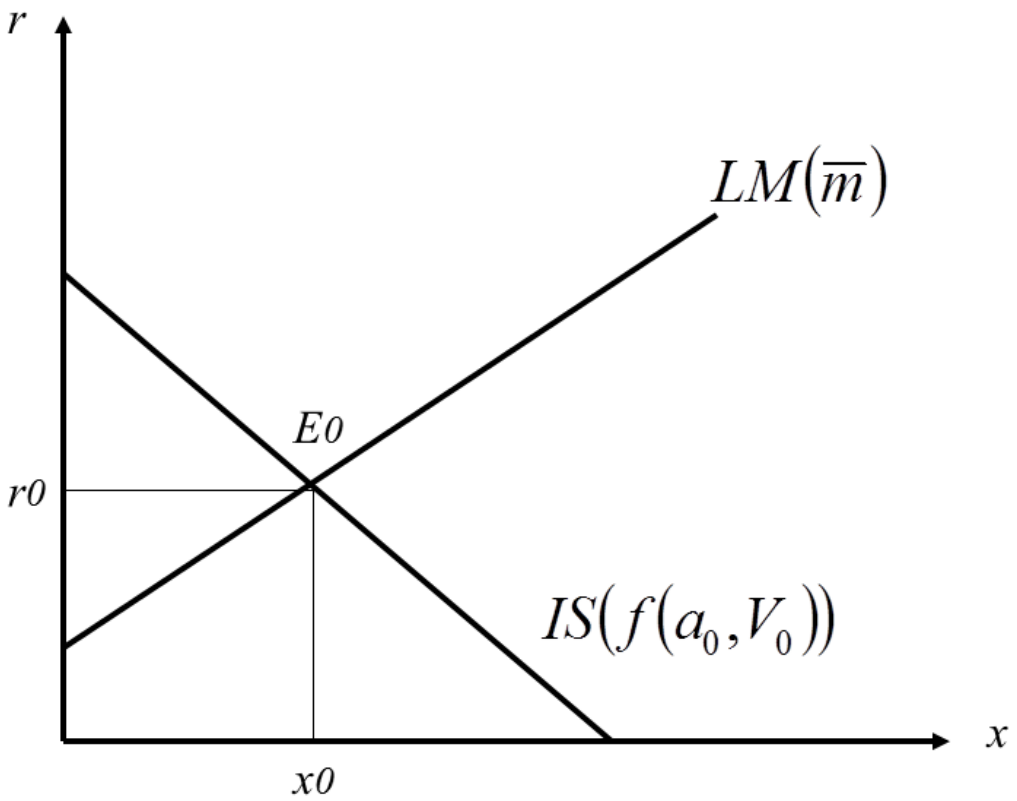
- We are now able to compute the EFP in the "long run" i.e when the distribution of the equity ratio has reached her long run equilibrium captured by the steady state cross sectional mean and variance:

$$f_0 \equiv \frac{\beta\alpha}{a_0}\left(1+\frac{V_0}{a_0^2}\right)$$

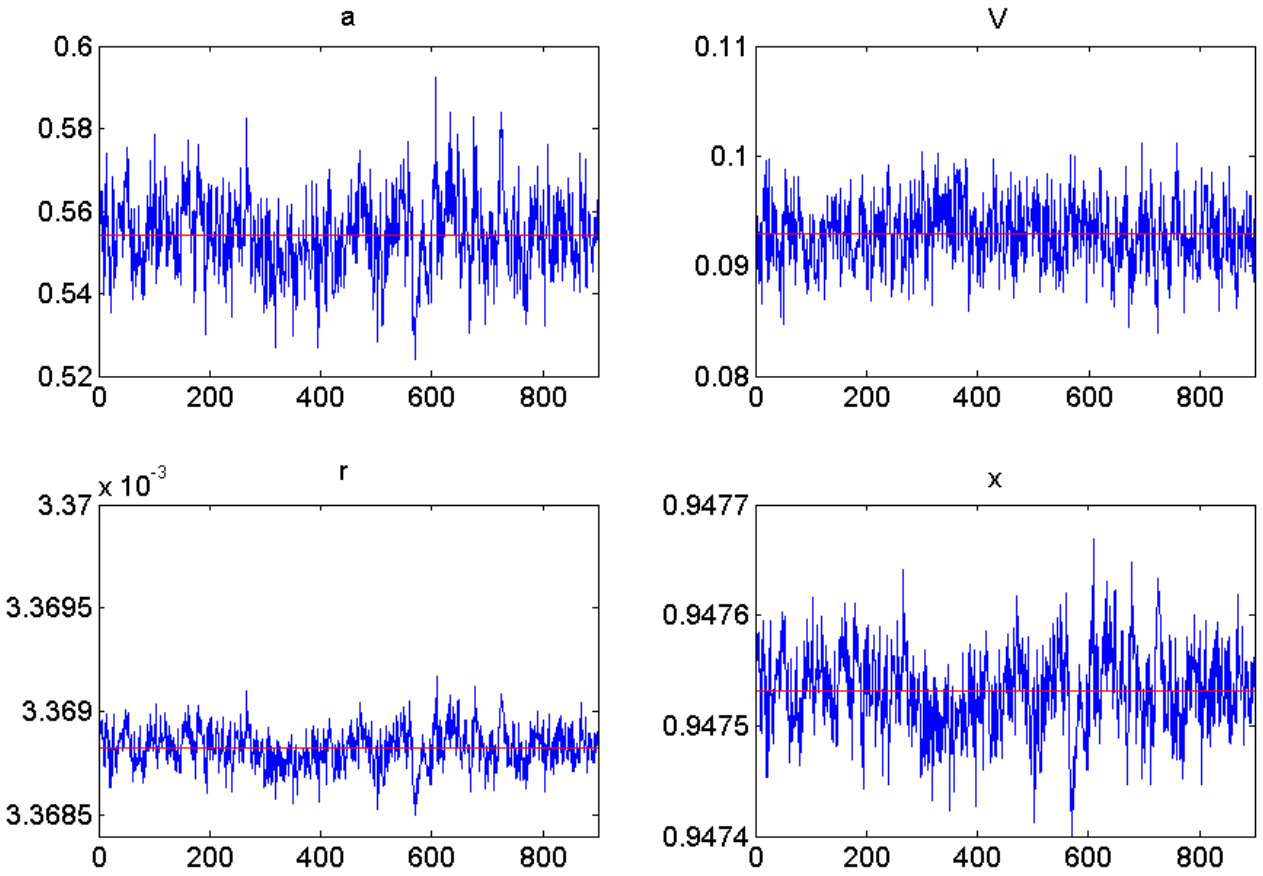
- This is the crucial datum we have to retrieve from the agent based model and plug into the reduced form of the macroeconomic model in order to compute the real interest rate and the output gap *in equilibrium* and *in the long run*:

$$\begin{aligned} r_0 &= \Gamma_0\left[\zeta w-\omega f_0\right] \\ x_0 &= \frac{1}{\omega}\left\{\bar{m}\Gamma_0\left[\zeta w-\omega f_0\right]-\sigma\right\} \end{aligned}$$

The "moments augmented" IS-LM model in steady state



A comparison with the time series of artificial data



A negative financial shock

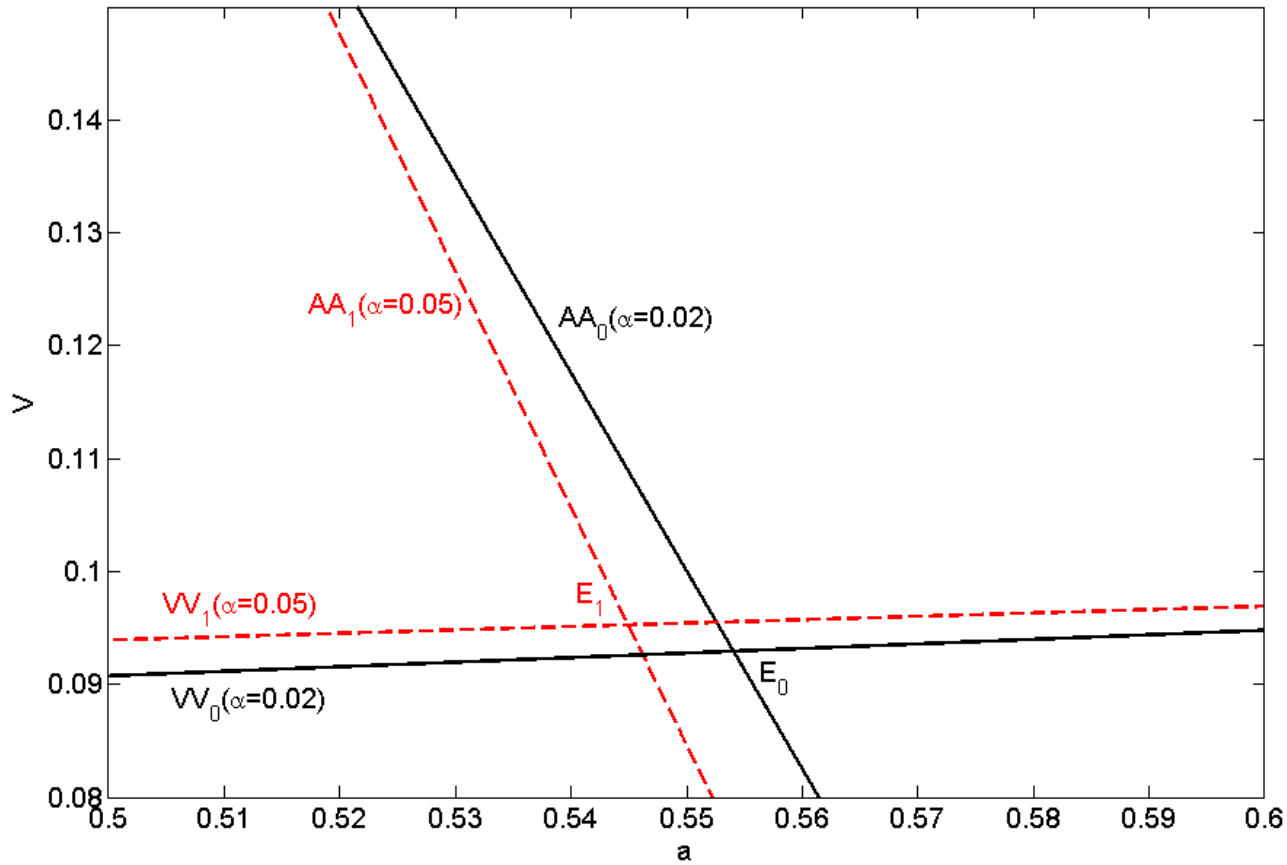
- *Negative financial shock*: exogenous increase of the bankruptcy threshold from $\alpha = 0.02$ to $\alpha' = 0.05$
- We generate a new set of artificial data running simulations as before but increasing the threshold
- We re-estimate the coefficients of the bivariate AR(1) process

Effects on the steady state distribution of the equity ratio

- From the new estimated coefficient we infer that the AA line shifts down and becomes steeper while the VV curve shifts up and becomes flatter
- If the shock is permanent the economy settles in the new steady state $E_1 = (a_1, V_1)$, characterized by a lower cross sectional mean of the equity ratio and a higher variance
- Therefore the equilibrium external finance premium will be higher

$$f_1 \equiv \frac{\beta\alpha'}{a_1}\left(1+\frac{V_1}{a_1^2}\right) > f_0 \equiv \frac{\beta\alpha}{a_0}\left(1+\frac{V_0}{a_0^2}\right)$$

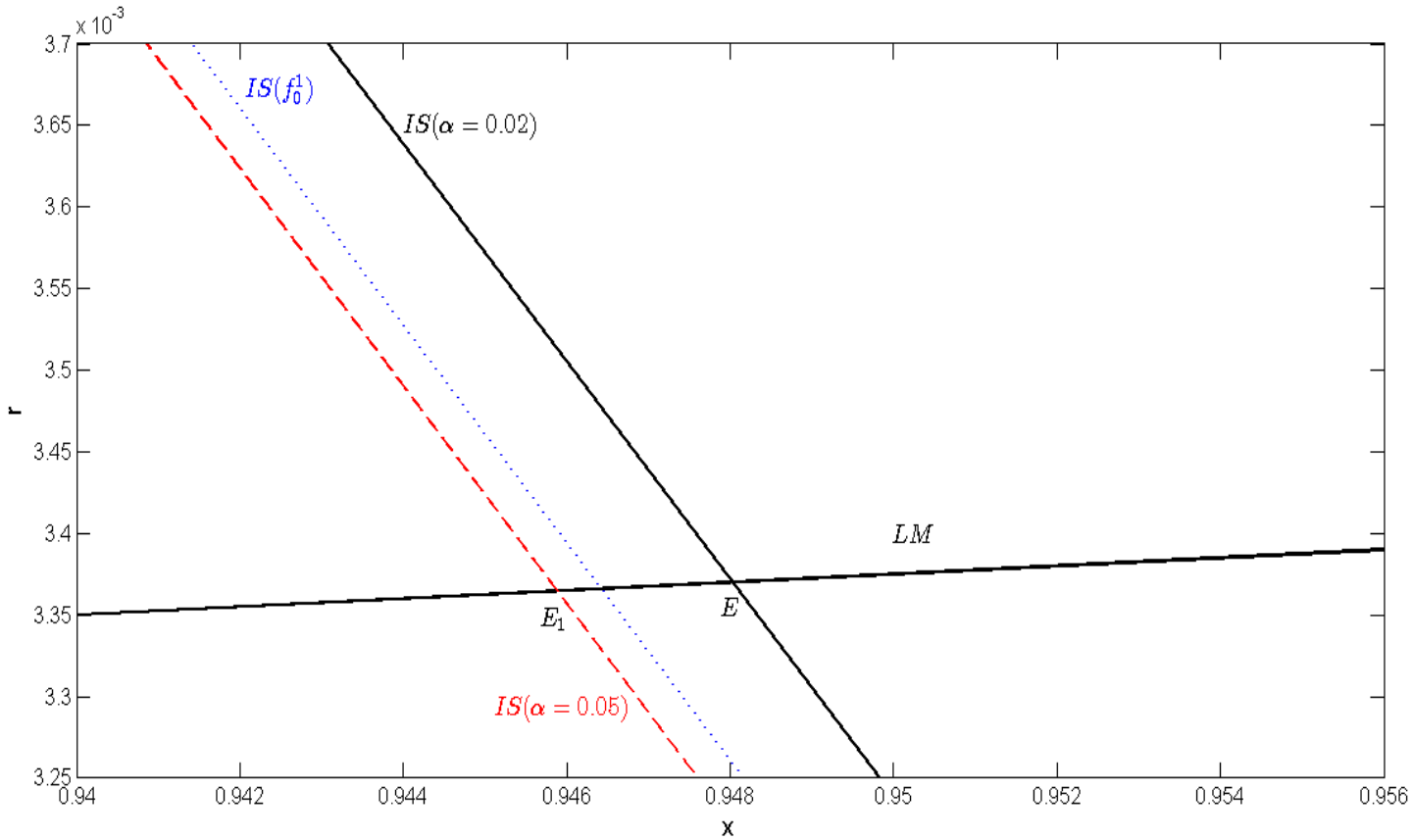
The phase diagram



Effects on the steady state interest rate and output gap

- The real interest rate and the output gap can be computed from the reduced form of the macroeconomic model: r_1, x_1
- It is clear that the negative financial shock makes both the output gap and the interest rate decrease: $r_1 < r_0, x_1 < x_0$

The moments augmented IS-LM model



The transmission mechanism: first round effect

- The first round effect of the shock is an increase of the probability of bankruptcy for each and every firm, given the original steady state distribution
- Hence the external finance premium (EFP) will increase on impact

$$f_0 \equiv \frac{\beta\alpha}{a_0}\left(1+\frac{V_0}{a_0^2}\right) \Rightarrow f_0' \equiv \frac{\beta\alpha'}{a_0}\left(1+\frac{V_0}{a_0^2}\right)$$

$$\Delta f_{1st} := f_0' - f_0 = \frac{\alpha' - \alpha}{\alpha} f_0$$

- This initial increase of the EFP makes the IS curve shift down along the LM curve (blue IS schedule)

The transmission mechanism: second round effect (financial amplification)

- The shock triggers a dynamic downward adjustment of the equity ratio for each and every firm: $\Longrightarrow$ the cross sectional mean of the equity ratio will go down (pushing up the EFP) and the variance will increase (pushing further up the EFP)
- The new increase of the EFP ($f_0' \rightarrow f_1$) will push the IS curve further down along the LM curve (red IS schedule)

$$\Delta f_{2nd} := f_1 - f_0' = \beta\alpha' \left[\left(\frac{1}{a_1} - \frac{1}{a_0} \right) + \left(\frac{V_1}{(a_1)^3} - \frac{V_0}{(a_0)^3} \right) \right]$$

Second round effect (cont'd)

- *Heterogeneity contributes to the magnification* of the initial shock
- Second round effect without heterogeneity:

$$\beta\alpha' \left(\frac{1}{a_1} - \frac{1}{a_0} \right)$$

- Contribution of heterogeneity to the second round effect:

$$\beta\alpha' \left(\frac{V_1}{(a_1)^3} - \frac{V_0}{(a_0)^3} \right)$$

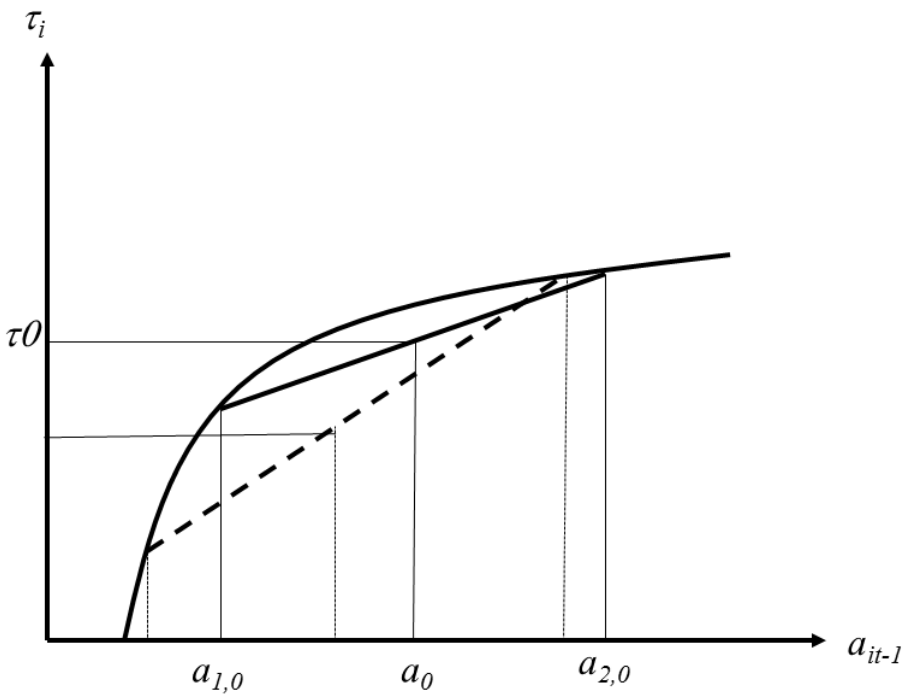
Second round effect (cont'd)

- In our simulation, the 2nd round $\left(7.4 \times 10^{-6}\right)$ effect is two orders of magnitude smaller than the 1st round effect $\left(1.4 \times 10^{-4}\right)$
- The contribution of heterogeneity, however, is half of the 2nd round effect
- The increase of dispersion will add to the increase of the EFP due to the reduction of the cross sectional mean of the equity ratio

What's going on...beyond the noise? A financial accelerator story

- The non-linear specification of the probability of bankruptcy makes the financial shock hit harder on the relatively fragile firms (those with a low equity ratio)
- These firms experience an increase in the probability of bankruptcy higher than the increase affecting robust firms.
- Hence their equity ratio will fall more than the equity ratio of the robust firms, increasing the variance.

What's going on?



Conclusions

- We have described – and applied – a methodology to build hybrid macroeconomic and agent based models
- The next step consists in building a NK-DSGE model with a Financial accelerator based on heterogeneous financial conditions
- Can we bring this model to the data?