Overlapping Portfolios and Financial Contagion

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Context

- **Financial networks**: networks of financial institutions (banks) with mutual relationships (e.g. Allen and Gale 00, Boss et al. 05, Iori et al. 08, E Santos and Cont 10...)

- **How can stress that originates in a part of the system be amplified and propagated to the whole system?** (e.g. Gai and Kapadia 10, Amini et al. 10, Georg 10, May and Arinaminpathy 10, May and Haldane 11, Arinaminpathy et al. 12, ...)

- **Many amplification mechanisms**: we focus on common asset holdings (overlapping portfolios)
Overlapping portfolios and Market Impact

- Market impact: prices respond to trades (e.g. Engle et al. 08, Bouchaud et al. 09)

- Portfolio liquidation $\rightarrow$ assets devaluation

- Banks with common assets are exposed to contagion
Related Literature

• Cifuentes, Shin and Ferrucci (2005): one asset common to all banks

• Beale et al. PNAS (2011): individual vs systemic risk

• Corsi, Marmi and Lillo, SSRN (2013): overlapping portfolios and financial innovation

• Huang, Vodenska, Havlin and Stanley, Scientific Reports (2013): empirical analysis of US commercial banks
• Random network: links are drawn randomly

• Large network: \( N, M \to \infty \), but finite \( N/M \)

• Sparse network: \( \mu_b \ll N, M \)
global overlap

local overlaps
A bank is solvent if its equity is positive, i.e. assets > liabilities
Leverage: banks borrow money to build portfolios;

We start with a system of solvent banks and depress the value of a random asset;

If a bank becomes insolvent, its portfolio of assets undergoes a fire-sale;

Market impact: fire-sales depress prices, which can cause other banks to fail;

Under what conditions do we observe global cascades of failures?
Relevant Parameters

- Average diversification, average degree of banks: \( \mu_b \)

- Crowding: \( N/M \)

- Leverage: \( \lambda = \frac{\text{assets}}{\text{equity}} \)
Homogeneous System

- The size of the balance sheet is the same for all banks;

- 80% of each banks’ total assets are invested in a portfolio of illiquid assets;

- Portfolio weights are uniform;

- Banks have the same initial leverage;
Some Definitions

• There is a global cascade of failures if a finite fraction of an infinite system goes bankrupt. In simulations, if at least 5% of banks go bankrupt.

• Contagion probability: probability of observing a global cascade.

• Conditional extent of contagion: average fraction of bankruptcies given that a global cascade occurs.
$N/M = 1 \quad \lambda = 20$

Diversification

Robust yet fragile
low diversification: disconnected network

higher diversification: network is well connected
low diversification: disconnected network

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Higher diversification: network is well connected
Robust yet fragile.

$N/M = 1 \quad \lambda = 20$

Probability and extent of contagion

Contagion window

Diversification
**Leverage**

- Increasing leverage makes the system more unstable;
- At fixed average degree, there is a critical leverage below which contagion probability is zero.
Branching Processes

one ancestor...

... generates $x$ offspring
(with $x$ a random variable)

global cascades occur with non-zero probability
if $E[x] > 1$
In our case

one bankrupted bank...

...causes $x$ other banks to fail

global cascades occur with non-zero probability if $E[x] > 1$
Stability Matrix

Probability that i fails given the failure of j:

$$B_{ij} = \text{Prob} \left[ \sum_{a=1}^{M} Q_{ia} p_a (1 - f_a(Q_{ja})) - E_i > 0 \right]$$

Number of banks of type h that fail if a bank of type k fails.

$$N_{hk} = N_h \sum_{a=1}^{M} P(h, k|a) F(h|k, a)$$

Compute the largest eigenvalue of the matrix to know about stability.
The analytic approach seems to underestimate the width of the contagion window, this is in part due to finite size effects.
Approximation

failures due to “siblings” are not accounted for
contagion probability is not monotonic in $N/M$ and $\mu_b$. Increasing leverage makes the system overall more unstable, but there is a minimum leverage for which contagion occurs.
Summary

- Overlapping portfolios and market impact as a contagion mechanism.
- Contagion probability is non-monotonic in the average diversification and the relative number of banks to assets (crowding).
- The system exhibits a robust yet fragile behavior.
- Analytical characterization of phase space.
Future Directions

- Calibration with real data
- Dynamics: prices and strategic behavior of banks
- Interaction between different contagion channels
theoretical work: mostly on interbank lending

empirical work: interbank lending networks are resilient
Are networks important at all?

Theoretical work: mostly on interbank lending

Empirical work: interbank lending networks are resilient
are networks important at all?

yes, they can amplify stress due to other mechanisms

All banks invest a fraction $c$ of their portfolio in a common asset.

External shock: the common asset is devalued.

- How many banks fail because of the initial shock?
- How many more fail because of counterparty loss?
Network of direct exposures can amplify shocks due to overlapping portfolios.
All banks invest a fraction $c$ of their portfolio in a common asset.

External shock: one bank fails.

When a bank fails its portfolios is liquidated and the common asset devalued.

How many banks fail because of fire sales?

How many more fail because of counterparty loss?
Interaction between contagion mechanisms

![Graph showing the interaction between contagion mechanisms. The graph compares the conditional extent of contagion with and without a network. The x-axis represents the fraction of common asset, ranging from 0 to 1. The y-axis represents the contagion probability, ranging from 0 to 0.5 on the left graph, and the conditional extent of contagion, ranging from 0 to 1 on the right graph. The red dashed line indicates the with network condition, and the blue line indicates the without network condition.](image-url)
Conclusions

• Simple model of overlapping portfolios and market impact as vector of contagion

• Interaction between different mechanisms of contagion
Thanks!

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More crowding shifts contagion window to the left
Initial failure of a random bank

different initial condition: initial bankruptcy of a random bank

- Global cascades occur in the same window of connectivities;
- Probability of contagion is different;
- If contagion occurs, the average size of global cascades is the same;
The analytic approach seems to underestimate the width of the contagion window, this is in part due to finite size effects.
Vulnerable cluster (in red): set of connected banks that go bankrupt if one member of the set fails.

- probability of hitting the cluster: random bank 3/5, random asset: 2/3
- once the cluster is hit the outcome is the same