The multiplex Structure of the Italian Interbank Market

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INET Workshop on Interlinkages and Systemic Risk

Ancona, July 5th 2013

The research leading to these results has received funding from the European Union VII Framework Programme FP7/2007-2013 under grant agreement CRISIS-ICT-2011-288501

The views expressed in the presentation are those of the authors only and do not involve the responsibility of the Bank of Italy

Credit markets as Multiplex network: Research Questions

Two banks can entertain many types of credit relations (e.g. overnight vs. long term, collateralized vs. uncollateralized contracts). A layer is defined by the set of contracts of the same type. The ensemble of all layers is the multiplex network. We wish to answer the following questions:

- Are network properties similar across layers?
- Are layers globally similar to each other?
- Are layers and their properties stable over time (even with respect to policy shocks)?

Dataset

- Source: monthly supervisory reports to the BoI
- A wide range of collateralized (secured) / uncollateralized (unsecured) debt instruments included
- Reclassified data w.r.t. to maturity:
  - overnight
  - short term ($t \leqslant$1Y)
  - long term ($t >$1Y)
- End of year data for the period 2008 - 2012
- Group Consolidation (selfloops)
- Transactions with CCPs and the Eurosystem not included
- **In this presentation we look at domestic data**

Stylized facts on Interbank networks

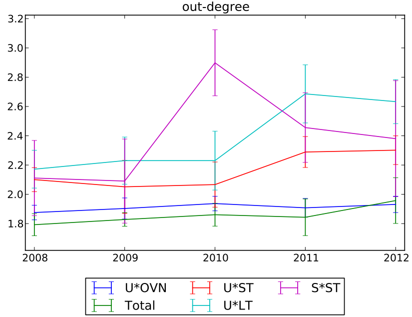
- Sparsity, low avg. distance
- Power-law distribution of strengths / degrees with small exponents ($\ll 3$)
- Disassortative mixing (Fricke *et al.*, 2013)
- Low Clustering (Boss *et al.*, 2004; Cont *et al.*, 2010; Iori *et al.*, 2007)
- Heterogeneous levels of reciprocity (Bech & Atalay, 2008)

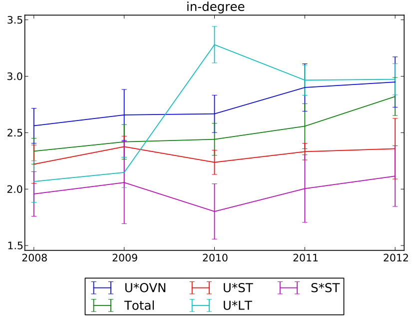
Basic Network Properties of Layers, 2012

Statistics	U*OVN	U*ST	U*LT	S*ST	Total
# of nodes	595	481	444	38	597
# of edges (net)	2,957	1,103	681	45	3,478
# of selfloops	66	34	24	18	72
Density	0.008	0.005	0.003	0.032	0.010
Avg. undir. distance	2.345	3.097	2.785	2.585	2.325
Out-degree assortativity	-0.4433	-0.4694	-0.4591	-0.0430	-0.4390
In-degree assortativity	-0.2270	-0.3492	-0.6993	-0.0442	-0.2471
Out-degree est. exponent	1.93	2.30	2.62	2.56	1.81
In-degree est. exponent	3.02	2.25	2.98	2.01	2.85
Out-weight est. exponent	1.63	1.75	2.22	1.46	2.02
In-weight est. exponent	2.11	1.82	1.97	1.35	1.96

U = Unsecured ; S = Secured; OVN = Overnight; ST = Short Term;
LT = Long Term

Maximum-Likelihood estimation of α in $p(x) \propto x^{-\alpha}$ (Clauset *et al.*, 2009)





Log-likelihood Ratio Test of Power-Law against Lognormal (Clauset *et al.*, 2009)

Layer	2008	2009	2010	2011	2012
U*OVN	-1.5975	-0.6155	-0.7908	-0.5973	-0.7747
U*ST	-0.4928	-2.4985	-0.3699	-0.1970	-0.4383
U*LT	-0.2803	-0.1650	-2.3005	-0.1321	-0.0672
S*ST	-0.4841	-1.5027	-0.2103	-0.0695	-0.5836
Total	-1.7883	-0.6418	-0.8288	-1.4540	-0.2354

Table: Out-degree distribution

Layer	2008	2009	2010	2011	2012
U*OVN	-1.5975	-0.6155	-0.7908	-0.5973	-0.7747
U*ST	-0.4455	-0.3426	-0.0668	-0.0343	-1.2056
U*LT	-0.1346	-0.6715	-0.2187	0.0131	-0.0168
S*ST	-2.1298*	-2.3327**	-0.7419	-2.3029	-1.2691**
Total	-0.3304	-0.0006	-0.0434	-0.1290	-0.3487

* 5% significant; ** 1% significant;

Table: In-degree distribution

Exponential Random Networks as null models

- Starting point: graph theory (Chung), statistical mechanics of networks (Park & Newman, 2002), further elaborations from Fagiolo,Garlaschelli & Squartini, Gallegati & Bargigli
- Random binary (fermionic) networks with a fixed expected degree distribution
- Links a_{ij} are non identical independent Bernoulli variables
- No need to estimate degree distributions, no exact constraint on degree values (microcanonical ensemble)
- Computational trade-off: parameters sometimes difficult to estimate but networks easy to simulate once parameters are obtained

Benefits from an economic viewpoint

- Strength distribution not a topological property, refers to economic structure (size) & behavior (supply / demand)
- Strengths and degrees should be independent in a perfectly diversified market, instead they are strongly correlated

Layers	2008	2009	2010	2011	2012
U*OVN	0.7498	0.6885	0.6945	0.6786	0.7013
U*ST	0.9056	0.9168	0.9045	0.8194	0.7828
U*LT	0.8625	0.8591	0.9711	0.9184	0.8981
S*ST	0.8824	0.8913	0.6649	0.8077	0.7502
Total	0.7666	0.7158	0.6808	0.5314	0.4905

Spearman correlation: out-degree vs. out-weight

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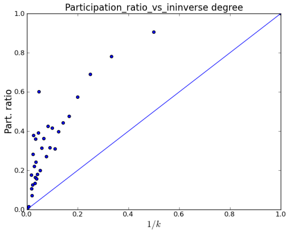

Total data, 2012

- Strengths and degrees distributions are determined by complex economic interaction, and may change abruptly over time

Topological properties of the domestic Italian interbank market

	2008	2009	2010	2011	2012
Largest strong component	445	471	469	472	444
(p-values)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Simulation average	330.5	354.9	353.4	364.0	336.8
Degree reciprocity	0.345	0.381	0.376	0.362	0.345
(p-values)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Simulation average	0.223	0.238	0.231	0.231	0.211
Undirected triangles	18,111	14,760	14,589	14,592	13,713
(p-values)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Simulation average	23,105	20,216	20,044	19,710	18,968

Table: Unsecured overnight

	2008	2009	2010	2011	2012
Largest strong component	24	11	65	173	164
(p-values)	(0.230)	(0.129)	(0.000)	(0.000)	(0.000)
Simulation average	21.2	16.4	38.4	113.3	109.5
Degree reciprocity	0.061	0.033	0.114	0.413	0.377
(p-values)	(0.448)	(0.260)	(0.000)	(0.000)	(0.000)
Simulation average	0.064	0.046	0.070	0.215	0.197
Undirected triangles	378	273	258	192	168
(p-values)	(0.004)	(0.005)	(0.000)	(0.000)	(0.000)
Simulation average	576.2	459.5	575.0	966.1	887.0

Table: Unsecured long-term

Exponential networks: Bosonic & sparse extensions

- Links in weighted or Bosonic exp. networks follow non identical independent geometric distributions
- It's possible to solve the Bosonic model with both the average strength and degree distributions fixed (Bianconi, 2008; Bargigli, 2013)
- But very difficult to estimate the parameters of this model
- Alternatively, we may suppose that w_{ij} is the product of two independent variables (Bernoulli and Poisson): much easier to estimate

Sparse Bosonic Network Realization

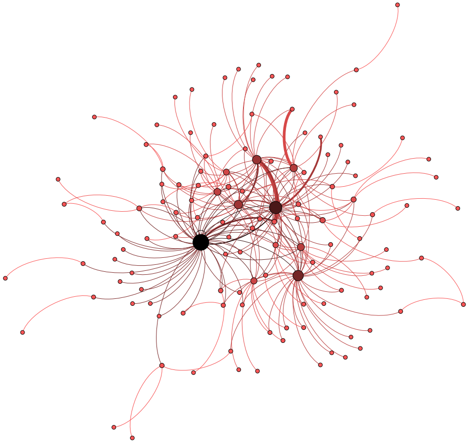

Figure: Unsecured long-term, 2009.
Note: The core - periphery division is determined by the degree distribution (Lip, 2011)

Strength-related properties of the Italian Interbank market

	2008	2009	2010	2011	2012
Weight reciprocity	0.404	0.167	0.186	0.421	0.286
(p-values)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Simulation average	0.047	0.009	0.016	0.0325	0.031

Table: Unsecured overnight

	2008	2009	2010	2011	2012
Weight reciprocity	0.007	0.000	0.000	0.000	0.017
(p-values)	(0.022)	(0.686)	(0.602)	(0.102)	(0.006)
Simulation average	0.001	0.000	0.001	0.000	0.002

Table: Unsecured short-term

Similarity Analysis of Layers: Measures

We opt for functions $\in [0, 1]$ with no assumptions on the underlining distribution (correlation excluded):

- Jaccard similarity for binary data:

$$J(\mathbf{p}, \mathbf{q}) = \frac{|\mathbf{p} \wedge \mathbf{q}|}{|\mathbf{p} \vee \mathbf{q}|} \quad (1)$$

- Cosine similarity for valued data:

$$\cos(\theta) = \frac{\mathbf{p} \cdot \mathbf{q}}{||p||||q||} \quad (2)$$

Steps

- We treat networks as r^2 dimensional vectors, where r is the size of the set of nodes spanning the two networks
- We determine this set by taking alternatively the intersection or the union of the nodes of each network
- We measure anti-correlation by computing similarity w.r.t. the vector obtained from the transpose adjacency or strength matrix
- Permutation test for statistical significance

Jaccard Similarity within Layers

	2008	2009	2010	2011
2009	0.4227*			
2010	0.4393*	0.4767*		
2011	0.2989*	0.2877*	0.4535*	
2012	0.2143*	0.2909*	0.2951*	0.3438*

* 1% significant

Table: Unsecured short-term (Intersection)

	2008	2009	2010	2011
2009	0.6833*			
2010	0.5674*	0.6531*		
2011	0.5389*	0.5943*	0.7156*	
2012	0.4964*	0.5394*	0.6398*	0.7210*

* 1% significant

Table: Unsecured overnight (Intersection)

Cosine Similarity within Layers

	2008	2009	2010	2011
2009	0.9898*			
2010	0.9890*	0.9906*		
2011	0.9813*	0.9899*	0.9937*	
2012	0.0078*	**0.0130***	**0.0578***	**0.1289***

* 1% significant

Table: Unsecured short-term (Intersection)

	2008	2009	2010	2011
2009	0.3304*			
2010	0.1098*	0.4038*		
2011	0.1498*	**0.4631***	0.5393*	
2012	0.1132*	**0.3666***	**0.4874***	**0.8021***

* 1% significant

Table: Unsecured overnight (Intersection)

Unsecured short-term: topological properties

	2008	2009	2010	2011	2012
Largest strong component	237	246	287	272	297
(p-values)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Simulation average	134.9	136.6	163.5	173.2	171.7
Degree reciprocity	0.419	0.400	0.422	0.432	0.453
(p-values)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Simulation average	0.184	0.175	0.185	0.2	0.198
Undirected triangles	1656	1632	1536	1059	837
(p-values)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Simulation average	3622.1	3460.0	3915.5	3047.2	2947.2

Jaccard similarity across Layers

	S*ST	U*OVN	U*LT
U*OVN	0.0096*		
U*LT	0.0112*	0.1906*	
U*ST	0.0067*	0.2060*	0.2255*

* 1% significant.

Table: Jaccard Similarity, 2012 (Union)

	S*ST	U*OVN	U*LT
U*OVN	0.0872*		
U*LT	0.1429*	0.2486*	
U*ST	0.0745*	0.2424*	0.2969*

* 1% significant.

Table: Jaccard Similarity, 2012 (Intersection)

Jaccard Similarity with the Transpose

	S*ST	U*OVN	U*LT
U*OVN	0.0084*		
U*LT	0.0070*	0.1680*	
U*ST	0.0038	0.1803*	0.2622*

* 1% significant.

Table: Jaccard Similarity, 2012 (Union)

	S*ST	U*OVN	U*LT
U*OVN	0.0764*		
U*LT	0.0847*	0.2178*	
U*ST	0.0412	0.2114*	0.3486*

* 1% significant.

Table: Jaccard Similarity, 2012 (Intersection)

Conclusions

- Network Properties of layers:
  - Significant topological properties: degree reciprocity, strong connectivity, clustering
  - Topological & strength-related properties not stable across layers (e.g. weight reciprocity), excepted clustering / triangles
- Global Comparison of layers:
  - Weak topological & weighted similarity across layers
  - Credit relationships not reciprocated at different layers
- Stability:
  - Topological properties of some layers change over time: degree distribution (U*LT), degree reciprocity (U*LT), triangles (U*LT & U*ST); other layers are very stable (U*OVN)
  - Less pronounced stability from similarity analysis, some radical changes become apparent only with this approach (U*ST)