When Micro Prudence increases Macro Risk: The Destabilizing Effects of Financial Innovation, Leverage, and Diversification

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joint work with

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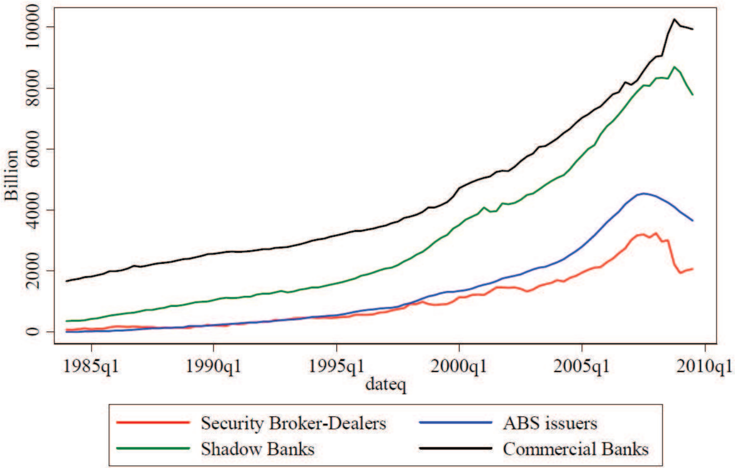
The Role of Credit and Financial Intermediaries (FIs)

- In most standard economic models, financial institutions (FIs) are viewed as passive players and credit does not have any macroeconomic effect.
- Yet, recent empirical work found: accelerations in credit supply (bank assets) is the key antecedent to financial crises (e.g. Schularick and Taylor 2012)
- These empirical results confirm that balance sheet dynamics of FIs, is the "endogenous engine" driving the boom-bust cycles and hence systemic risk.
- Adrian Shin (2010) quote: *"balance sheet aggregates such as total assets and leverage are the relevant financial intermediary variables to incorporate into macroeconomic analysis"*

Balance Sheet expansion of FIs



Note: Shadow banks are ABS issuers, finance companies, and funding corporation
Source: Board of Governors of the Federal Reserve

from Adrian and Shin (2010)

Related literature

Our paper tries to combine several strands of literature:

- on the impact of capital requirements on the behavior of FIs (Danielsson et al., 2004,2009; Adrian & Shin,2009; Adrian et al., 2011);
- on the effects of diversification and overlapping portfolios on systemic risk (Tasca & Battiston, 2012; Caccioli et al., 2012)
- on the risks of financial innovation (Brock et al. 2009, Haldane & May, 2011)
- on distressed selling and its impact on the market price dynamics (Kyle & Xiong, 2001; Cont & Wagalath, 2011, Thurner et al., 2012; Caccioli et al., 2012)
- on the determinants of balance sheet dynamics of FIs and credit supply (Stein 1998; Bernanke & Gertler 1989; Bernanke, Gertler, & Gilchrist, 1996,1999; Kiyotaki & Moore, 1997)

Contribution: propose a simple model that, by combining these different streams of literature, provides full analytical quantification of the links between micro prudential rules and macro prudential outcomes

The portfolio choice

- For simplicity, we assume that FIs adopt a simple investment strategy: equally weighted portfolio of m randomly selected investment (out of M)
- we consider the existence of “costs of diversification" c reflecting the presence of transaction costs, firms specialization and other types of frictions.
- With r_L the avg interest rate on liabilities the portfolio expected return is $\mu - r_L$,
- FI maximizes portfolio returns under VaR constraints.

$$VaR = \alpha \sigma_p A \leq E.$$

with σ_p the holding period volatility, A asset of bank i, and α a constant,

The investment set

The investment set:

- collection of risky investment $j = 1, ..., M$
- FIs, correctly perceive that each risky investment entails both an idiosyncratic (diversifiable) risk component and a systematic (undiversifiable) risk component,

  $$\sigma_i^2 = \sigma_s^2 + \sigma_d^2$$

  where

  - σ_s^2 is the systematic risk
  - σ_d^2 is the diversifiable risk component.

- Hence, the expected mean and volatility per dollar invested in the portfolio chosen by a given institution are μ and

  $$\sigma_p = \sqrt{\sigma_s^2 + \frac{\sigma_d^2}{m}}$$

The portfolio optimization

Then, facing cost of diversification and VaR constraints, FI chooses the total asset A (E is sticky) and diversification m which max their portfolio returns.

$$\max_{A,m} \quad A(\mu - r_L) - \tilde{c}m \qquad \text{s.t.} \quad \alpha A\sqrt{\sigma_s^2 + \frac{\sigma_d^2}{m}} \leq E.$$

Dividing by E and with $c = \frac{\tilde{c}}{E}$, the max can be written in terms of the leverage $\lambda = \frac{A}{E}$,

$$\max_{\lambda,m} \quad \lambda(\mu - r_L) - cm \qquad \text{s.t.} \quad \alpha\lambda\sqrt{\sigma_s^2 + \frac{\sigma_d^2}{m}} \leq 1.$$

$\rightarrow$ chooses the optimal leverage $\lambda^* = \frac{A^*}{E}$ and m^* which max ROE under the VaR

Squaring the constraint the Lagrangian can be written as

$$L = \lambda(\mu - r_L) - cm - \frac{1}{2}\gamma\left(\alpha^2\lambda^2\left(\sigma_s^2 + \frac{\sigma_d^2}{m}\right) - 1\right).$$

where γ is the Lagrange multiplier for the VaR constraint.

Optimal leverage and diversification

$$\text{F.O.C.} \quad \Rightarrow \quad \lambda^* = \frac{1}{\gamma}\,\frac{1}{\alpha^2}\,\frac{\mu - r_L}{\sigma_p^2} \quad \text{with Lagrange multiplier} \quad \gamma = \frac{1}{\alpha}\,\frac{\mu - r_L}{\sigma_p}$$

- The optimal leverage is

$$\lambda^* = \frac{1}{\alpha\sqrt{\sigma_s^2 + \frac{\sigma_d^2}{m}}} = \frac{1}{\alpha\sigma_p}$$

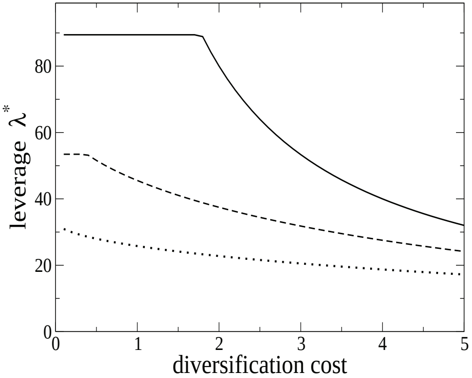
- The optimal level of diversification is

$$m^* = \frac{\sqrt{\gamma}\alpha\lambda\sigma_d}{\sqrt{2c}} = \lambda\sigma_d\sqrt{\frac{\alpha}{2c}\,\frac{\mu - r_L}{\sigma_p}}$$

Bottom line:

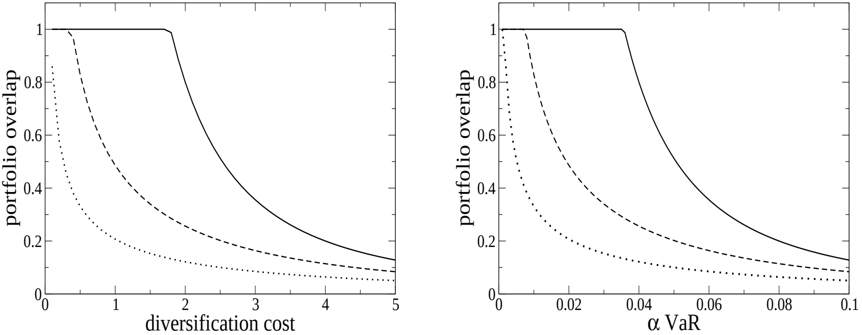
- leverage λ is an inverse function of the portfolio volatility σ_p
- portfolio size m is an inverse function of diversification costs c

Diversification cost and optimal leverage



parameters are: $M = 20$, $\alpha = 0.05$, $\mu - r_L = 0.8$, $\sigma_d = 1$.
We then choose σ_s equal to 0 (solid line), 0.3 (dashed line), and 0.6 (dotted line).

Diversification cost and portfolio overlap



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We then choose σ_s equal to 0 (solid line), 0.3 (dashed line), and 0.6 (dotted line).

Leverage targeting and balance sheet adjustments

Initial position

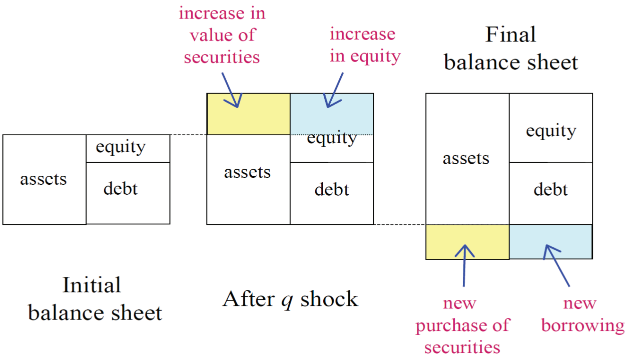
Assets	Liabilities
Asset 100	Debt 90
	Equity 10

Asset growth

Assets	Liabilities
Asset 101	Debt 90
	Equity 11

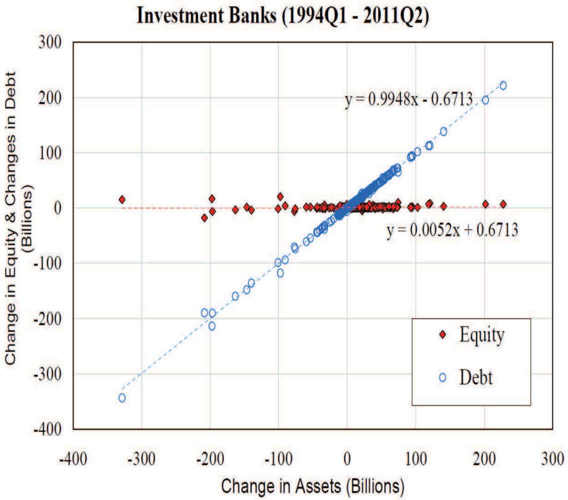
Leverage adjustment

Assets	Liabilities
Asset 110	Debt 99
	Equity 11



from Adrian and Shin (2010)

Balance sheet adjustments: empirical evidence



from Adrian, Colla, and Shin (2010)

Dynamics of asset with portfolio rebalancing

At the beginning of each investment period FIs rebalance their portfolio by the difference between the desired amount of asset $A^*_{j,t} = \lambda E_{j,t}$ and the actual one $A_{j,t}$

$$\Delta R_{j,t} \equiv A^*_{j,t} - A_{j,t} = \lambda E_{j,t} - A_{j,t},$$

By defining realized portfolio return $r^p_{j,t}$, can be rewritten as

$$\Delta R_{j,t} = (\lambda - 1) r^p_{j,t} A^*_{j,t-1}$$

$\Rightarrow$ any P&L from the investments portfolio $r^p_{j,t} A^*_{j,t-1}$ results in a change of FI asset value amplified by the target leverage (for $\lambda > 1$).

$\Rightarrow$ VaR induces a perverse demand function: buy if $r^p_{j,t} > 0$, sell if $r^p_{j,t} < 0$

$\Rightarrow$ positive feedback

Dynamics of investments demand

The aggregate demand of asset i will be simply the sum of the individual demands of the FIs who picked asset i in their portfolio.

$$D_{i,t} = \sum_{j=1}^{N} I_{\{i \in j\}} \frac{1}{m} \Delta R_{j,t} \approx \sum_{j=1}^{N} I_{\{i \in j\}} (\lambda - 1) r_{j,t}^{p} \frac{A_{j,t-1}^{*}}{m}$$

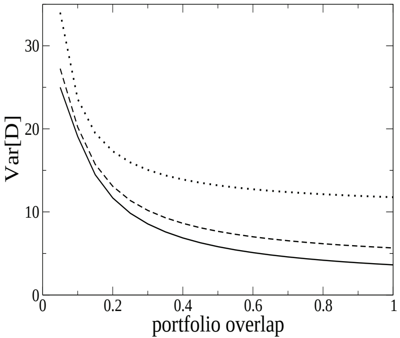
where $I_{\{i \in j\}}$ is 1 if asset i is in the portfolio of institution j and zero otherwise.

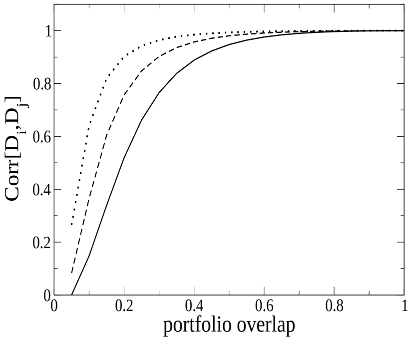
Considering total assets approximately the same across FIs, $A_{j,t-1}^{*} \simeq A_{t-1}^{*}$, demand of investment i can be approximated as

$$D_{i,t} \approx (\lambda - 1) \frac{A_{t-1}^{*}}{m} \frac{N}{M} \left(r_{i,t} + \frac{m-1}{M-1} \sum_{k \neq i} r_{k,t} \right)$$

Note: it can be shown that demand correlation between two assets $\rho(D_i, D_k) \underset{m \to M}{\to} 1$

Portfolio overlap and demand variance & correlation





Parameters are $M = 20$, $N = 100$, and $\sigma_d = 1$.
We then choose σ_s equal to 0 (solid line), 0.3 (dashed line), and 0.6 (dotted line).

Risky asset dynamics with endogenous feedbacks I

With rebalancing feedbacks, the return process is now made of 2 components

$$r_{i,t} = \underbrace{e_{i,t-1}}_{\text{endogenous}} + \underbrace{\varepsilon_{i,t}}_{\text{exogenous}}$$

We assume that the exogenous component has a multivariate factor structure

$$\varepsilon_{i,t} = \underbrace{f_t}_{\text{factor}} + \underbrace{\epsilon_{i,t}}_{\text{idiosyncratic}}$$

uncorrelated and distributed with mean 0 and constant volatility, σ_f and σ_ϵ (the same for all investments).

Thus, the variance of the exogenous component of the risky investment i is

$$V(\varepsilon_i) = \sigma_f^2 + \sigma_\epsilon^2$$

Risky asset dynamics with endogenous feedbacks II

Assuming a linear price impact function the endogenous component becomes

$$e_{i,t} = \frac{D_{i,t}}{\gamma_i C_{i,t}}$$

where

- γ_i is the market liquidity of asset i
- $C_{i,t} = \sum_{j=1}^{N} I_{\{i \in j\}} \frac{A^*_{j,t-1}}{m} \approx \frac{N}{M} A^*_{t-1}$ is a proxy for market cap

Substituting D, r, and C, we obtain the following VAR(1) for e_t

$$e_t = \mathbf{\Phi} \left(e_{t-1} + \varepsilon_t \right)$$

where $\mathbf{\Phi} \equiv (\lambda - 1) \quad \mathbf{\Gamma}^{-1} \quad \mathbf{\Psi}$ with

$$\underset{M \times M}{\mathbf{\Gamma}} = \begin{bmatrix} \gamma_1 & 0 & \dots & 0 \\ 0 & \gamma_2 & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & \gamma_M \end{bmatrix}, \qquad \underset{M \times M}{\mathbf{\Psi}} = \begin{bmatrix} \frac{1}{m} & \frac{1}{m}\frac{m-1}{M-1} & \dots & \frac{1}{m}\frac{m-1}{M-1} \\ \frac{1}{m}\frac{m-1}{M-1} & \frac{1}{m} & \dots & \frac{1}{m}\frac{m-1}{M-1} \\ \vdots & & \ddots & \vdots \\ \frac{1}{m}\frac{m-1}{M-1} & \frac{1}{m}\frac{m-1}{M-1} & \dots & \frac{1}{m} \end{bmatrix}.$$

Multivariate return dynamics

The VAR(1) dynamics

$$e_t = \mathbf{\Phi}\,(e_{t-1} + \varepsilon_t)$$

is dictated by the eigenvalues of the matrix

$$\mathbf{\Phi} \equiv (\lambda - 1)\mathbf{\Gamma}^{-1}\mathbf{\Psi}.$$

Being the max eigenvalue of $\mathbf{\Psi}$ equal 1 $\forall m$, we have:

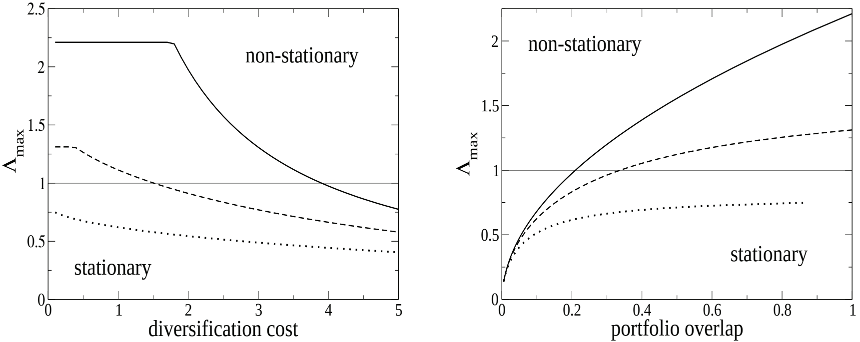
$$\Lambda_{\max} \approx (\lambda - 1)\ \ \overline{\gamma^{-1}}$$

where $\overline{\gamma^{-1}}$ is the average of all the γ_i^{-1}.

$\Rightarrow$ the max eig depends on leverage and on the average illiquidity of the assets.

When $\Lambda_{\max} > 1$, the return processes become non-stationary and explosive

Max eigenvalue, diversification cost and portf overlap



parameters are: $M = 20$, $\alpha = 0.05$, $\mu - r_L = 0.8$, $\gamma = 40$, and $\sigma_d = 1$.
We then choose σ_s equal to 0 (solid line), 0.3 (dashed line), and 0.6 (dotted line).
The horizontal solid line shows the condition $\Lambda_{max} = 1$,

Alternative representation of endogenous dynamics

We can write,

$$m\boldsymbol{\Psi} = \begin{bmatrix} 1 & \frac{m-1}{M-1} & \cdots & \frac{m-1}{M-1} \\ \frac{m-1}{M-1} & 1 & \cdots & \frac{m-1}{M-1} \\ \vdots & & \ddots & \vdots \\ \frac{m-1}{M-1} & \frac{m-1}{M-1} & \cdots & 1 \end{bmatrix} = (1-b)\boldsymbol{I} + b\boldsymbol{\iota}\boldsymbol{\iota}' \qquad \text{with} \quad b = \frac{m-1}{M-1}.$$

Thus, the endogenous component of an individual investment i becomes

$$e_{i,t} = \underbrace{(1-b)\ a_i(e_{i,t-1} + \varepsilon_{i,t})}_{\text{idiosyncratic comp}} + \underbrace{b\ Ma_i(\bar{e}_{t-1} + \bar{\varepsilon}_t)}_{\text{common average comp}} \quad \text{with} \quad a_i = \frac{\lambda - 1}{m\gamma_i}.$$

Moreover, assuming all investments have the same liquidity, we can show:

- the average process $\bar{e}_t$ is an AR(1) (systemic component)
- the distance from the avg $\Delta e_{i,t} \equiv e_{i,t} - \bar{e}_t$ is an AR(1) (idiosyncratic)

$\Rightarrow$ the endogenous return dynamics can be seen as a multivariate "ARs around AR"

Endogenous Variance & Covariance formulas

Thanks to this representation we can explicitly compute the variance and covariances of endogenous components $e_{i,t}$

$$V(e_{i,t}) = -\frac{(\lambda-1)^2\Big(m^2\Big(\sigma_\epsilon^2\big((\lambda-1)^2-\gamma^2(M-1)\big)+\sigma_f^2\big((\lambda-1)^2-\gamma^2(M-1)^2\big)\Big)+2m\Big(M\Big(\sigma_\epsilon^2\big(\gamma^2-(\lambda-1)^2\big)-(\lambda-1)^2\sigma_f^2\Big)-\gamma^2\sigma_\epsilon^2\Big)+M\Big(M\Big(\sigma_\epsilon^2\big((\lambda-1)^2-\gamma^2\big)+(\lambda-1)^2\sigma_f^2\Big)+\gamma^2\sigma_\epsilon^2\Big)\Big)}{\left(\gamma^2-(\lambda-1)^2\right)\left(m^2\left(\gamma^2(M-1)^2-(\lambda-1)^2\right)+2(\lambda-1)^2 mM-(\lambda-1)^2M^2\right)}$$

$$Cov(e_{i,t}, e_{j,t}) = -\frac{(\lambda-1)^2\left(m^2\left(\sigma_f^2\left((\lambda-1)^2-\gamma^2(M-1)^2\right)-\gamma^2(M-2)\sigma_\epsilon^2\right)-2m\left((\lambda-1)^2M\sigma_f^2+\gamma^2\sigma_\epsilon^2\right)+M\left((\lambda-1)^2M\sigma_f^2+\gamma^2\sigma_\epsilon^2\right)\right)}{\left(\gamma^2-(\lambda-1)^2\right)\left(m^2\left(\gamma^2(M-1)^2-(\lambda-1)^2\right)+2(\lambda-1)^2mM-(\lambda-1)^2M^2\right)}$$

and show that:

- $\uparrow$ leverage $\rightarrow$ $\uparrow$ both var and cov of $e_{i,t}$
- $\uparrow$ diversification $\rightarrow$ $\downarrow$ var and $\uparrow$ cov
- Both $\rightarrow$ $\uparrow$ correlations
- $Corr(e_{i,t}, e_{j,t}) \xrightarrow[m \to M]{} 1$

Return Var-Cov with rebalancing feedbacks

The feedback induced by portfolio rebalancing, introduces a new endogenous component in the variances and covariances of individual and portfolio returns

- individual returns:

$$V(r_{i,t}) = \underbrace{V(e_{i,t})}_{\text{endogenous}} + \underbrace{V(\varepsilon_{i,t})}_{\text{exogenous}} \qquad \text{with} \quad V(\varepsilon_{i,t}) = \sigma_f^2 + \sigma_\epsilon^2$$

$$Cov(r_{i,t}, r_{j,t}) = \underbrace{Cov(e_{i,t}, e_{j,t})}_{\text{endogenous}} + \underbrace{\sigma_f^2}_{\text{exogenous}}$$

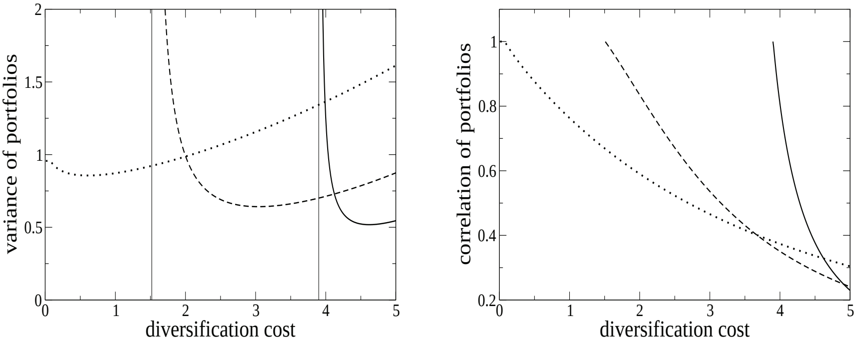
- portfolio returns:

$$V(r_t^p) = \underbrace{\frac{V(e_{i,t})}{m} + \frac{m-1}{m} Cov(e_{i,t}, e_{j,t})}_{\text{endogenous}} + \underbrace{\sigma_f^2 + \frac{\sigma_\epsilon^2}{m}}_{\text{exogenous}}$$

$$Cov(r_{h,t}^p, r_{k,t}^p) = \underbrace{V(\bar{e})}_{\text{endogenous}} + \underbrace{V(\varepsilon_{M,t})}_{\text{exogenous}} \qquad \text{with} \quad V(\varepsilon_{M,t}) = \sigma_f^2 + \frac{\sigma_\epsilon^2}{M}$$

$$V(r_t^M) = \underbrace{\frac{1}{1 - \Lambda_{\max}^2}}_{\text{“variance multiplier"}} V(\varepsilon_{M,t})$$

Portfolio variance & correlation vs diversification cost



Parameters are: $M = 20$, $\alpha = 0.05$, $\mu - r_L = 0.8$, $\gamma = 40$, and $\sigma_d = 1$.
We then choose σ_s equal to 0 (solid line), 0.3 (dashed line), and 0.6 (dotted line).

Bank asset dynamics

The dynamics of the rebalanced bank asset $A^*_{j,t}$, can be written as

$$A^*_{j,t} = \lambda E_{j,t} = \lambda \left(E_{j,t-1} + r^p_{j,t} A^*_{j,t-1} \right) = A^*_{j,t-1} + \lambda r^p_{j,t} A^*_{j,t-1}$$

thus, the relative change of the bank i total asset $r^A_{i,t}$ is simply given as

$$r^A_{j,t} \equiv \frac{A^*_{j,t} - A^*_{j,t-1}}{A^*_{j,t-1}} = \lambda r^p_{j,t}.$$

Therefore, the var-cov of the relative change of bank assets $r^A_{j,t}$ are simply

$$V(r^A_{j,t}) = \lambda^2 V(r^p_{j,t}) \qquad Cov(r^A_{h,t}, r^A_{k,t}) = \lambda^2 Cov(r^p_{h,t}, r^p_{k,t}),$$

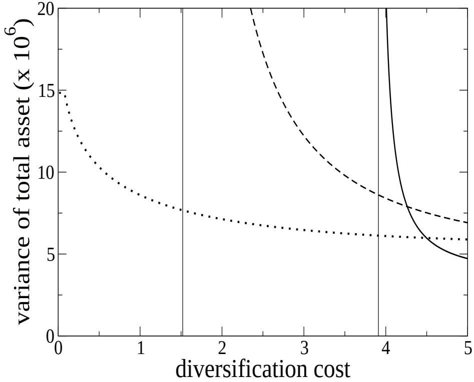
We can finally compute the variance of the total asset of the whole banking sector

$$V\left(\sum_{j=1}^{N} r^A_{j,t} \right) = \lambda^2 V\left(\sum_{j=1}^{N} r^p_{j,t} \right)$$

with

$$V\left(\sum_{j=1}^{N} r^p_{j,t} \right) \xrightarrow[m \to M]{} \frac{N^2 V(\varepsilon_{M,t})}{1 - \Lambda_{\max}}.$$

Variance of bank total asset and diversification cost



parameters are: $M = 20$, $\alpha = 0.05$, $\mu - r_L = 0.8$, $\gamma = 40$, $\sigma_d = 1$, and $N = 100$.
We then choose σ_s equal to 0 (solid line), 0.3 (dashed line), and 0.6 (dotted line).
The vertical lines indicate where the variance of total asset diverges

Systemic risk

- We can show that correlation between FI portfolio returns $\rho_p \xrightarrow[m \to M]{} 1$
- The total systematic (exogenous and endogenous) component is $s_t = \bar{e}_t + f_t$. The portfolio return distribution conditioned on a systematic shock s_t^{shock} is

$$r_{i,t}^p | s_t^{shock} \sim N\left(s_t^{shock}, \ \frac{\sigma_d^2}{m}\right).$$

$\Rightarrow$ probability of default of a FI given a systematic shock s_t^{shock} is

$$\begin{aligned} PD_{i,t-1} &= P\left(\left[r_{i,t}^p | s_t^{shock}\right] \leq -\alpha\sqrt{\sigma_s^2 + \frac{\sigma_d^2}{m}}\right) \\ &= \Phi\left(\frac{-\alpha\sqrt{\sigma_s^2 + \frac{\sigma_d^2}{m}} - s_t^{shock}}{\sqrt{\frac{\sigma_d^2}{m}}}\right) \xrightarrow[m \to M, \ M \to \infty]{} 1 \quad \forall \ s_t^{shock} < -\alpha\sigma_s, \end{aligned}$$

$\Rightarrow$ *robust yet fragile* behavior emerges

Bottom line: diversification tends to increase the probability of a systemwide failures.

Systemic risk (cont's)

- If endogenous components is not accounted for $\Rightarrow$ underestimation of risk, $\Rightarrow$ under capitalization of the banking sector $\Rightarrow$ higher system fragility.
- the practice of estimating var-cov of risky assets from past data, automatically considers both the exogenous and endogenous components.
- However, var-cov now depend on the level of diversification and leverage:
  - in periods $\uparrow$ leverage $\Rightarrow$ historical volatility underestimate future risk
  - in periods $\downarrow$ leverage $\Rightarrow$ historical volatility overestimate future risk

  $\rightarrow$ theoretical support for countercyclical capital requirements
- Finally, a negative realization of the factor f_t, now triggers a sequence of portfolio rebalances causing the price of all risky assets to decay for several periods. Being

$$r_t \quad = \quad e_{t-1} + \boldsymbol{\iota} f_t + \epsilon_t = \boldsymbol{\Phi} r_{t-1} + \boldsymbol{\iota} f_t + \epsilon_t, \quad \rightarrow \quad \text{also a VAR(1)}$$

the h-period cumulative mean return conditioned on systematic shock f_t^{shock} is

$$E\left[r_{t:t+h} | f_t = f_t^{shock}\right] \approx (\boldsymbol{I} - \boldsymbol{\Phi})^{-1} \boldsymbol{\iota} f_t^{shock}.$$

Introduction of financial innovation: summary

High costs of diversification $c \Rightarrow$ small diversification $m \Rightarrow$ heterog. portfolios and P&L $\Rightarrow$ individual feedbacks weak and uncoordinated.

Introduction of financial innovation makes: $\downarrow c \quad \uparrow m \quad \downarrow \sigma_p \quad \uparrow \lambda$

Hence we have:

1) Increase in leverage $\lambda \Rightarrow$ increases risk exposure

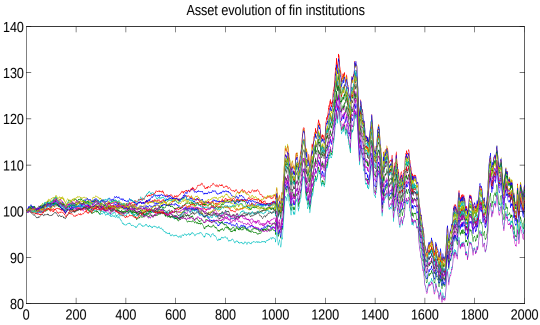
2) Increase in diversification $m \Rightarrow$ increases correlations

3) Increase in λ and $m \Rightarrow$ increases endogenous feedback $\Rightarrow \uparrow$ var, cov & corr

So, individual reaction more aggressive (due to higher leverage) and more coordinated (due to higher correlation) $\Rightarrow$ aggregate feedback between prices and total asset

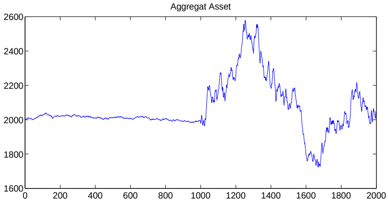
$\Rightarrow$ makes bank total asset A more erratic $\Rightarrow$ liquidity and funding booms and bursts

Simulation results: simulated structural break



Structural Break at 1000:

1) low diversification and leverage
2) high diversification and levarage



Summary & conclusions

i. the feedback between investment prices and bank asset induced by portfolio rebalancing leads to a multivariate VAR process whose max eigenvalue depends on the degree of leverage and average illiquidity of the assets;

ii. both the variance and correlation of individual investments monotonically increase with reduction in diversification costs; $+$ "variance multiplier" of mkt vol

iii. the relation between the portfolio variance and diversification costs is non-monotonic

iv. the endogenous feedback makes historical estimation of var-cov to be overestimated during periods of increasing leverage and underestimated during periods of deleveraging $\Rightarrow$ rationale for countercyclical capital requirements;

v. reduction in diversification costs, by increasing the strength and coordination of individual feedbacks, increases the variability of bank total asset, which governs the supply of credit and liquidity to financial system