Input-Output-based measures of systemic influence

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Outline

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Motivation

- Crisis highlighted importance of interlinkages
- Shift from *micro*- to *macro*-prudential approach to banking regulation and supervision
- ESRB, Dodd-Frank (FSOC), SSM
- Need to better understand how interconnectedness might affect the diffusion of stress through the system
- Indicators of systemic influence are of high importance in the current debate on reforming the financial system
    - "More systemically important institutions should be subject to stricter prudential requirements"

Context and taxonomy

- Notion of systemic risk is closely linked to that of externalities; systemic is a risk whose consequences are not confined to an individual financial institution but extend beyond it: to other financial institutions, to the real domestic economy, or to the global economy, etc.
- Explosion in research (see Bisias et al. (2012))
- Broadly speaking, there are two classes of indicators of systemic influence
  - Asset prices distributions and correlations across institutions, assets and time (Adrian and Brunnermeier (2011), Acharya et al. (2011) among others; some convincing criticisms: Danielsson et al. (2011) and Löffler and Raupach (2013))
  - Balance sheet interlinkages, network measures

Our approach

- Belongs to the second class
  - Key: Matrix of interbank lending/borrowing positions
- Both new and old
- Builds on a well-established and time-honored strand of literature
  - Easy to understand and explain
  - Economic intuition
- Should be seen as complementary to other measures

Outline

General set-up

- Banking system composed of n banks, each of which
  - Collects deposits (d_j) and equity (e_j)
  - Lends to non-bank customers (l_j)
  - Lends to (a_{ji}) and borrows from (a_{ij}) other banks
- Balance sheet of bank j

$$e_j + d_j + a_{1j} + \ldots + a_{nj} = a_{j1} + \ldots + a_{jn} + l_j \tag{1}$$

- Matrix representation (aggregate across n banks)

$$\mathbf{e} + \mathbf{d} + \mathbf{A}_M'\mathbf{i} = \mathbf{A}_M\mathbf{i} + \mathbf{l} \tag{2}$$

where $\mathbf{A}_M$ is the matrix of interbank bilateral positions

General set-up - The Leontief inverse

- Let $\mathbf{q} \equiv$ vector with total bank assets/liab.: $\mathbf{q} = \mathbf{e} + \mathbf{d} + \mathbf{A}'_M\mathbf{i} = \mathbf{A}_M\mathbf{i} + \mathbf{l}$
$\Longrightarrow$ $\hat{\mathbf{q}}$: corresponding diagonal matrix (i.e. $\hat{\mathbf{q}}\mathbf{i} = \mathbf{q}$)
- Then the r.h.s of (2) can be written in the following form:

$$\mathbf{q} = \mathbf{A}_M\hat{\mathbf{q}}^{-1}\hat{\mathbf{q}}\mathbf{i} + \mathbf{l} = \mathbf{A}\mathbf{q} + \mathbf{l} \tag{3}$$

$\mathbf{A} = \mathbf{A}_M\hat{\mathbf{q}}^{-1}$: matrix of interbank positions in which each *column* is divided by the total assets of the *borrowing* bank

$\Longrightarrow$ Columns of $\mathbf{A}$ express the ratio of funding from other banks to total funding

- Relation between loans and total assets given by the Leontief inverse $\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$:

$$\mathbf{q} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{l} = \mathbf{B}\mathbf{l} \tag{4}$$

General set-up - An alternative formulation

- Alternative: "supply-side" model (Ghosh (1958), Miller & Blair (2009))

$$\mathbf{q} = \underbrace{\mathbf{e} + \mathbf{d}}_{\equiv \mathbf{v}} + \mathbf{A}_M'\mathbf{i} = \mathbf{v} + \mathbf{A}_M'\mathbf{i} \tag{5}$$

- Transposing and operating:

$$\mathbf{q}' = \mathbf{v}' + \underbrace{\mathbf{i}'\hat{\mathbf{q}}}_{\mathbf{q}'}\underbrace{\hat{\mathbf{q}}^{-1}\mathbf{A}_M}_{\mathbf{O}}$$

$$\Longrightarrow \mathbf{q}' = \mathbf{v}'\underbrace{(\mathbf{I} - \mathbf{O})^{-1}}_{\mathbf{G}} \tag{6}$$

$\mathbf{O} = \hat{\mathbf{q}}^{-1}\mathbf{A}_M$: matrix of "output" coefficients, i.e. matrix of interbank positions in which each *row* is divided by the total assets of the *lending* bank

I-O model and how we use it

1. Production of sector j has two distinct effects: upstream/downstream

   $\Longrightarrow$ Rasmussen (1956) and Hirschman (1958): identification of key industrial sectors (those characterised by many linkages with others)

2. Changes in the matrix $\mathbf{A}$ (or $\mathbf{O}$)

   $\Longrightarrow$ "fields of influence" (Sonis and Hewings (1989,1991,2009))

3. Transmission of risk in interrelated infrastructural systems

   $\Longrightarrow$ "inoperability" (Haimes (2009)); "hypothetical extraction" (Cella (1984))

Outline

First case - Introduction

- Write (2) as $\mathbf{l} = (\mathbf{A}'_M - \mathbf{A}_M)\mathbf{i} + \mathbf{e} + \mathbf{d} \equiv \mathbf{s}$
- Shock coming from funding side, say deposits of bank 1
  - First impact: balance sheet loss for bank 1 ($\mathbf{s}$, $\mathbf{q}$)
  - Second round: effect spread via first column of $\mathbf{A}$
  - ... Final effect: on each bank given by $\mathbf{Bi}_1$, total system effect by $\mathbf{i}'\mathbf{Bi}_1$

"Backward" / "Forward" linkages

- Rasmussen-Hirschman "backward" index for bank j: $h_{b_j} = \mathbf{i}'\mathbf{B}\mathbf{i}_j$. Normalize by the "intensity" of $\mathbf{B}$ and multiply by n

$$\bar{h}_{b_j} = n\frac{\mathbf{i}'\mathbf{B}\mathbf{i}_j}{\mathbf{i}'\mathbf{B}\mathbf{i}} \tag{7}$$

- "Power of dispersion": strength with which initial shock is dispersed through the system
- A similar "forward" index, based on supply-driven model instead:

$$\bar{h}_{f_j} = n\frac{\mathbf{i}_j'\mathbf{G}\mathbf{i}}{\mathbf{i}'\mathbf{G}\mathbf{i}} \tag{8}$$

- "Sensitivity of dispersion": how sensitive is bank j to a shock hitting all banks

First case - summing up

Table: Classification based on backward and forward linkages

		$\bar{h}_{f_j}$	
		< 1	> 1
$\bar{h}_{b_j}$	< 1	Weakly linkages oriented bank (generally independent)	Forward linkages oriented bank (dependent on demand for its funds)
	> 1	Backward linkages oriented bank (dependent on funds from others)	Key bank (generally dependent)

Second case - Fields of Influence

- Shock may come from interbank flows themselves
  $\Longrightarrow$ "field of influence" (Sonis and Hewings (1989,1991,2009))
- Say a_{ij} changes, then change in $\mathbf{B}$ is given by the $n \times n$ matrix

$$\mathbf{F}(i,j) = (\mathbf{B}\mathbf{i}_i)\left(\mathbf{i}_j'\mathbf{B}\right) = \mathbf{b}_i\mathbf{b}_j' \qquad (9)$$

$\mathbf{b}_i = \mathbf{B}\mathbf{i}_i$ and $\mathbf{b}_j' = \mathbf{i}_j'\mathbf{B}$ are resp. the i^{th} column and j^{th} row of $\mathbf{B}$

- **Case 1**: a bank, say j, which is cut financing by all others
  $\Longrightarrow$ Unit decline in all elements of column j of $\mathbf{A}$ (except $a_{jj} = 0$)
- The (normalized) effect on the system's total assets is given by

$$\bar{f}_{c_j} = n\frac{\mathbf{i}'\left(\sum_{i\neq j}\mathbf{F}(i,j)\right)\mathbf{i}}{\mathbf{i}'\left(\sum_{j=1}^{n}\sum_{i\neq j}\mathbf{F}(i,j)\right)\mathbf{i}} \qquad (10)$$

Row and Total Field of Influence

- **Case 2**: a given bank i cuts financing to all others
  $\Longrightarrow$ Unit decline in all elements of row i of $\mathbf{O}$ (except $a_{ii} = 0$)
- Strength of the systemic effects of bank i cutting credit to all other banks, relative to the mean of the system ("row field of influence"):

$$\bar{f}_{r_i} = n \frac{\mathbf{i}' \left(\sum_{j \neq i} \mathbf{F}(i,j) \right) \mathbf{i}}{\mathbf{i}' \left(\sum_{i=1}^{n} \sum_{j \neq i} \mathbf{F}(i,j) \right) \mathbf{i}} \tag{11}$$

- **Case 3**: combination of the previous two cases, i.e. the bank in question, say j, sees sources of interbank funds restricted and cuts its own supply to other banks ("total field of influence"):

$$\bar{f}_{t_j} = \frac{\bar{f}_{c_j} + \bar{f}_{r_j}}{2} \tag{12}$$

Third case - Total Linkage Effect

- So far size not explicitly taken into account, yet is obviously important
- Define another index that measures the total systemic effect of a complete "cut-off" of bank j from the interbank system
- "Total linkage effect" (Cella(1984)); "Hypothetical extraction method"

$$\bar{t}_j = \frac{\mathbf{i}'\mathbf{q} - \mathbf{i}'\mathbf{q}^{-j}}{\mathbf{i}'\mathbf{q}} = \frac{\mathbf{i}'(\mathbf{B} - \mathbf{B}^{-j})\mathbf{I}}{\mathbf{i}'\mathbf{q}} \tag{13}$$

where $\mathbf{B}^{-j} = (\mathbf{I} - \mathbf{A}^{-j})^{-1}$ and $\mathbf{A}^{-j}$ is the matrix obtained from $\mathbf{A}$ by setting all elements of the j^{th} row and j^{th} column to zero

I-O measures - Summary

Table: Indicators of systemic influence

Index	Description	Interpretation
$\bar{h}_{b_j}$	Backward R-H	System effect of a unitary liquidity shock in bank j
$\bar{h}_{f_j}$	Forward R-H	Effect on bank j of a unitary system-wide liquidity shock
$\bar{f}_{c_j}$	Column FoI	System effect of a unitary cut of interbank lending by bank j
$\bar{f}_{r_j}$	Row FoI	System effect of a unitary cut of interbank lending to bank j
$\bar{f}_{t_j}$	Total FoI	System effect of a unitary cut of all int. transactions by bank j
$\bar{t}_j$	Total linkage	System effect of a cut-off of bank j from interbank market

Outline

Empirical application to European banks

- Dataset of interbank exposures for 53 large European banks, presented in Alves et al. (2013)
- Anonymized snapshot of interbank exposures as of end 2011, compiled by national regulators within a joint EBA-ESRB statistical project
- Focus on total assets exposure (credit claims + debt securities + other assets), short and long term
- Combine with data on bank-specific characteristics proxying size, business model, risk, robustness and liquidity

Network of large European banks

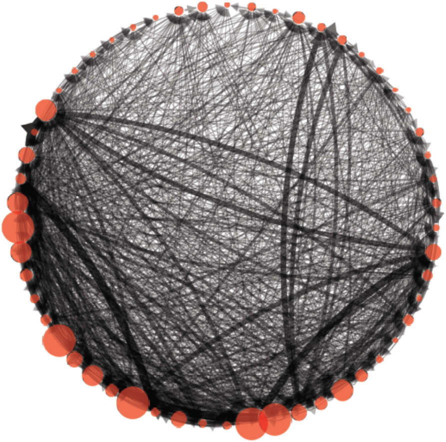

Figure: Network visualization - Total assets exposure

Note: Total assets exposure includes *credit claims*, *debt securities* and *other assets* exposures, both short and long term.

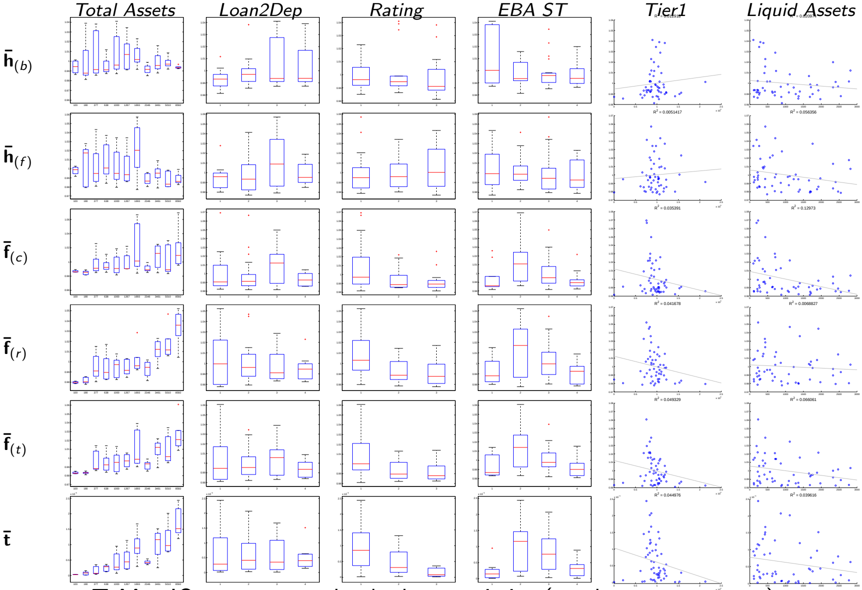

Table: IO measures vs bank characteristics (total assets exposure)

Note: *Total Assets*: Total assets (normalized). *Loan2Dep*: Loan to deposits ratio; 1: less than 120%, 2: between 120% & 150%, 3: between 150% & 200%, 4: more than 200%. *Rating*; 1: AA/A, 2: BBB, 3: Speculative. *EBA ST*: EBA 2011 Stressed Capital Ratios; 1: less than 6%, 2: between 6% & 7%, 3: between 7% & 9%, 4: more than 9%. *Tier1*: Tier 1 capital in % of assets. *Liquid Assets*: Liquid assets in % of assets.

	Total vs Assets	Total Long vs Short	Long Total vs Assets	Short Total vs Assets	Assets Long vs Short
$\bar{\mathbf{h}}_{(b)}$					
$\bar{\mathbf{h}}_{(f)}$					
$\bar{\mathbf{f}}_{(c)}$					
$\bar{\mathbf{f}}_{(r)}$					
$\bar{\mathbf{f}}_{(t)}$					
$\bar{\mathbf{t}}$					

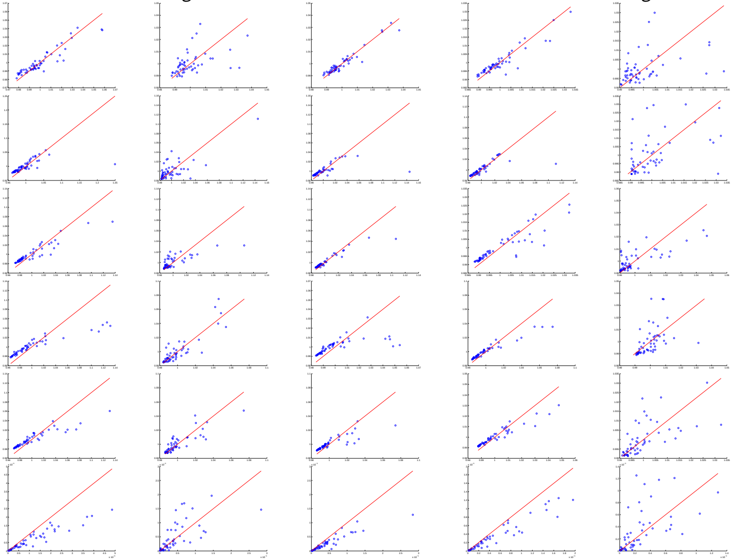

Table: Comparison of IO measures for different datasets

Note: 45 degree line in red. To be read "(x-axis) vs (y-axis)" (for a given IO indicator). The different datasets indicate different matrices of exposures.

Outline

Relation to network centrality measures

Robert Solow (1952)

"It is by no means an infrequent occurrence in economics that theories which are "about" different things turn out to be formally similar or even identical"

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"It is by no means an infrequent occurrence in economics that theories which are "about" different things turn out to be formally similar or even identical"

- IO special kind of directed and weighted network that must obey some boundary flow conditions (McNerney (2009))
- IO $\Longrightarrow$ matrix representation of the economy that allows for a complete picture of inter-sectoral relationships
- Link is under-researched in the literature (Olsen (1992), McNerney (2009), Blöchl et al. (2011), Acemoglu et al. (2012))
- First paper to explore link between linkage and centrality measures

Centrality measures considered

- $\mathbf{c}_{(in)}$, $\mathbf{c}_{(out)}$: most basic and intuitive centrality measures and attempt to capture network activity. *In-strength* centrality ($\mathbf{c}_{(in)}$) measures for node i the number of ties directed to it, using the value of the links as weights; similarly for *out-strength* centrality ($\mathbf{c}_{(out)}$)
- $\mathbf{c}_{(cl\text{-}in)}$, $\mathbf{c}_{(cl\text{-}out)}$: *closeness* centrality measures, for node i, the shortest path between i and all other nodes reachable from it, averaged across all other nodes. Aims to assess independence of nodes
- $\mathbf{c}_{(bw)}$: *betweennes* centrality gauges how often a given node lies in the shortest path between all other pairs of nodes. Importance of the node in terms of its role in the flow of activity in the network.
- $\mathbf{c}_{(lev)}$, $\mathbf{c}_{(rev)}$: *eigenvector* centrality identifies the importance of nodes by the Perron eigenvector; a node is central to the extent that it is connected to other nodes which are themselves central
- Together they capture the three main aspects of centrality: connectivity, proximity, betweenness (see Borgatti and Everett (2006))

IO and network centrality measures

Table: Relationship between I-O and network measures (R^2)

	$\mathbf{C}(in)$	$\mathbf{C}(out)$	$\mathbf{C}(cl\text{-}in)$	$\mathbf{C}(cl\text{-}out)$	$\mathbf{C}(bw)$	$\mathbf{C}(lev)$	$\mathbf{C}(rev)$
$\bar{\mathbf{h}}_{(\mathbf{b})}$	+	+	-	-	-	+	+
$\bar{\mathbf{h}}_{(\mathbf{f})}$	+	+	+	-	-	+	+
$\bar{\mathbf{f}}_{(\mathbf{c})}$	+	+	+	-	+	+	+
$\bar{\mathbf{f}}_{(\mathbf{r})}$	+	-	-	+	+	+	+
$\bar{\mathbf{f}}_{(\mathbf{t})}$	+	+	+	-	+	+	+
$\bar{\mathbf{t}}$	+	+	+	-	+	-	+

$<$ 10%	10%-19%	20%-39%	40%-59%	60%-79%	80%-89%	90%-100%

Note: When compared to $\bar{\mathbf{h}}_{(\mathbf{f})}$ and $\bar{\mathbf{f}}_{(\mathbf{r})}$, the centrality measures were computed based on the output matrix $\mathbf{O}$, otherwise they were computed based on the input matrix $\mathbf{A}$.

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Table: Relationship between I-O and network measures (R^2)

	$\mathbf{c}_{(in)}$	$\mathbf{c}_{(out)}$	$\mathbf{c}_{(cl\text{-}in)}$	$\mathbf{c}_{(cl\text{-}out)}$	$\mathbf{c}_{(bw)}$	$\mathbf{c}_{(lev)}$	$\mathbf{c}_{(rev)}$
$\bar{\mathbf{h}}_{(\mathbf{b})}$	+ ♣	+	-	-	-	+	+
$\bar{\mathbf{h}}_{(\mathbf{f})}$	+	+♣	+	-	-	+	+
$\bar{\mathbf{f}}_{(\mathbf{c})}$	+	+	+	-	+	+	+
$\bar{\mathbf{f}}_{(\mathbf{r})}$	+	-	-	+	+	+	+
$\bar{\mathbf{f}}_{(\mathbf{t})}$	+	+	+	-	+	+	+
$\bar{\mathbf{t}}$	+	+	+	-	+	-	+

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Note: When compared to $\bar{\mathbf{h}}_{(\mathbf{f})}$ and $\bar{\mathbf{f}}_{(\mathbf{r})}$, the centrality measures were computed based on the output matrix **O**, otherwise they were computed based on the input matrix **A**.

♣ - Proposition: direct positive relation between $\mathbf{h}_{(b)}$ ($\mathbf{h}_{(f)}$) and $\mathbf{c}_{(in)}$ ($\mathbf{c}_{(out)}$)

IO and network centrality measures

Table: Relationship between I-O and network measures (R^2)

	$\mathbf{c}_{(in)}$	$\mathbf{c}_{(out)}$	$\mathbf{c}_{(cl\text{-}in)}$	$\mathbf{c}_{(cl\text{-}out)}$	$\mathbf{c}_{(bw)}$	$\mathbf{c}_{(lev)}$	$\mathbf{c}_{(rev)}$
$\bar{\mathbf{h}}_{(\mathbf{b})}$	+ ♣	+	-	-	-	+ ♠	+
$\bar{\mathbf{h}}_{(\mathbf{f})}$	+	+ ♣	+	-	-	+	+ ♠
$\bar{\mathbf{f}}_{(\mathbf{c})}$	+	+	+	-	+	+	+
$\bar{\mathbf{f}}_{(\mathbf{r})}$	+	-	-	+	+	+	+
$\bar{\mathbf{f}}_{(\mathbf{t})}$	+	+	+	-	+	+	+
$\bar{\mathbf{t}}$	+	+	+	-	+	-	+

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Note: When compared to $\bar{\mathbf{h}}_{(\mathbf{f})}$ and $\bar{\mathbf{f}}_{(\mathbf{r})}$, the centrality measures were computed based on the output matrix $\mathbf{O}$, otherwise they were computed based on the input matrix $\mathbf{A}$.

♣ - Proposition: direct positive relation between $\mathbf{h}_{(b)}$ ($\mathbf{h}_{(f)}$) and $\mathbf{c}_{(in)}$ ($\mathbf{c}_{(out)}$)

♠ - $\mathbf{h}_{(b)}$ ($\mathbf{h}_{(f)}$) converges to left (right) Perron eigenvector of matrix $\mathbf{A}$ ($\mathbf{O}$) (Dietzenbacher (1992))

Outline

Concluding remarks

- New measures capturing different aspects of systemic relevance
- Grounded on well-established and time-honored tradition in economic analysis
- Take balance sheet as building block
- Rich and sensible economic interpretation of systemic relevance
- Straighforward implementation and easy to explain
- Possible links to existing measures
- Useful tool for analysis of systemic importance in policy institutions

THANK YOU!

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