Pollution abatement versus spillover effects: is urban toll a relevant fiscal tool?

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Summary: Using an urban economic model with an endogenous centre, we prove to what extent policies aimed at reducing pollution generated by working households' commuting trips involve a better control of urban sprawl as a positive side-effect. After the eco-tax has been levied by a local planner in the form of a kilometric urban toll, spillover effects which incited firms to group together in the Central Business District (*CBD*) keep on working as a positive externality. Under certain conditions, they are even strengthened.

Keywords: *urban economics, land use, environmental tax, pollution, urban toll, urban sprawl, accessibility, commuting, spillover effects.*

JEL classification: R48

1. Introduction

Congestion, noise, road accidents, pollution…these are some of the well-known external effects generated by the high use of single occupant cars in large cities. Such impacts have been aggravated in many developed countries by urban sprawl. Indeed, even in a larger territory, car use is increasing due to fixed travel times, and providing efficient public transport on an extended territory is complicated even if governed by a single local government. Urban pollution models have been developing since the seventies in favour of local intervention (Robson, 1976, Kanemoto, 1980). These models promote environmental policy tools destined to internalize the damage. Among them is the urban toll as an example of a pigouvian tax. It has been applied for example in London since 2003 or in Stockholm since 2006, but its origins date back at least to Singapore in 1975.

Negative externalities are numerous within the city. Lucas (2001) presents the major ones. If urban toll is initially levied to finance an infrastructure, it is later justified by the correction of a congestion externality, and more recently pollutant emissions (as in Milan since 2008). Different forms of pollution pricing (kilometric toll, area-based toll or urban toll cordons, Derycke, 1997) as well as differentiated amounts of toll (London compared with Singapore) denote that no theoretical guide really exists, maybe apart from the useful tools of the Boiteux report (2001) that advocates levying the tax to the value of marginal damage. Additive or alternative tools are tractable as well. Land use taxation may occur in the damage correction (Song and Zenou, 2006). Transport subsidies should encourage less pollutant public transport solutions (Wrede, 2000, Borck and Wrede, 2008, Su and de Salvo, 2008); other authors advocate tradable emission quotas applicable to car-drivers as an original solution (Raux, 2007).

Households which drive to work generate pollution which local government may have to correct as a negative externality. We consider firms' location choices as workplaces for households. Economic literature has argued that spillover effects exist independently, which nearby firms take advantage of through their production process. They constitute a positive agglomeration effect. In a competition framework, firms should isolate to keep a specific market area. Hotelling (1929) thus argued an opposite process, in which two sellers in a monopolistic competition framework tend to get closer to acquire a greater global demand. Anderson *et al.* (1992) clarified that idea: when concentration of production

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emerges, good differentiation is sufficient to relax price competition. Thus it appears that preference for diversity of consumers encourages the concentration rather than the dispersion of work places in a central area (the Central Business District, *CBD*). The formation of cities is therefore explained (Henderson, 1974, O'Hara 1977).

Urban sprawl emerges *in fine* as a "net of production concentration" effect (Duranton, 1997, Fujita and Thisse, 2003). Urban sprawl was first examined in the eighties (Brueckner and Fansler, 1983, Nelson, 1985), and was fully explored a decade later (Anas *et al.*, 1998, Brueckner, 2001). It has recently been described in several theoretical (Wu, 2006, Anas and Pines, 2008) and empirical papers (Anas and Xu, 1999, Song and Zenou, 2006)[1]. Agglomeration effects were introduced in theoretical models with negative externalities as well, in particular congestion (Arnott, 2007, Thissen *et al.*, 2010). In those schemes where a second source of distortion emerges, levying a tax to the marginal damage level is no longer the one that maximizes social welfare.

Our work is set in this original framework: in a medium-sized city, we seek to establish to what extent a kilometric urban toll works against pollution as a negative externality. We also consider the joined impact on the control of urban sprawl. Agglomeration effects are considered as an initial positive externality in favour of firms. In an urban economic model with an endogenous centre (Ogawa and Fujita, 1980), firms take advantage of sufficiently intense spillover effects which favoured a complete grouping in the *CBD*. The working household's trade-off between land use and accessibility (as the distance from the *CBD*) can then be introduced. The household budget constraint is raised by an eco-tax applied to his commuting trips. His residential location will depend on this trade-off. This choice will lead to a longer or shorter distance from the *CBD* and associated pollutant emissions. Firms freely choose labour and land quantities for production; as a result, the level of spillover effects of which they take advantage is determined.

The model shows, under certain conditions that will be defined, that introducing an environmental tax in the form of an urban kilometric toll leads to a simultaneous drop in pollutant emissions, a better control of urban sprawl and, which is more surprising, in strengthened agglomeration effects between firms in the *CBD.*

The paper is organized as follows: in section 2, starting from an urban economic model with an endogenous centre where agglomeration effects are revealed, the analytical framework is presented. Land use is explicitly introduced as an endogenous variable regarding the two types of agents. In section 3, a negative pollution externality combined with working households' commuting trips is considered. In section 4, implementing a kilometric urban toll, this negative externality is corrected.

2. Agglomeration effects in an urban monocentric model with an endogenous centre

2.1. Residential equilibrium of households

Ogawa and Fujita's (1980) urban monocentric pattern with an endogenous centre is first considered. We do not claim to demonstrate the monocentric pattern again: we consider it is realistic when large European cities are regarded, and we postulate the conditions for which this pattern is stable. Some other patterns exist under alternative conditions: we briefly present them in Appendix 1.

In a linear city where land is homogenous everywhere, N households (N exogenous) choose their residential location. The quantity of land available at every location x is equal to 1. M firms (M endogenous) locate on the interior part of a $[-r_0,r_0]$ segment, the Central Business District (*CBD*), such as at every distance x from the centre, $x\in\left[-r_0,r_0\right]$, *m(x)* firms set up. Households set up on the exterior parts of the segment $[-r_{max},-r_0]$ and $[r_0,r_{max}]$, r_{max} being the city boundary. At every distance x from the *CBD*, the number of households is equal to *n(x)*. Beyond $-r_{max}$ and r_{max} agricultural land use is noticed outside of the city (*Figure 1):*

[1] Urban sprawl is not the only geographical phenomenon working at the households' scale either: if several income classes are distinguished (Hartwick *et al.*, 1976), specific amenities may on the contrary lead well-off households to prefer the *CBD*, as may be noticed in European cities.

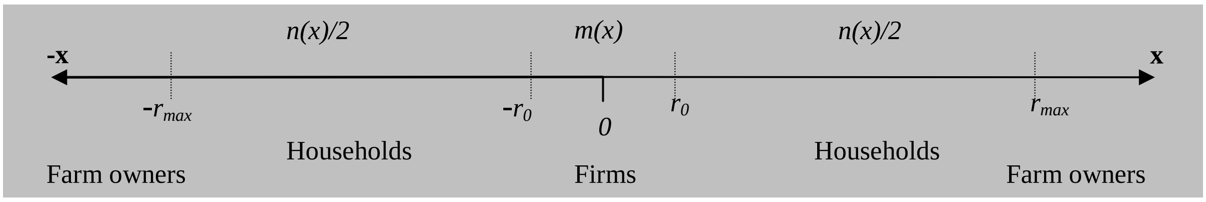

Figure 1 – The linear monocentric city with an endogenous centre

In the rest of the paper, without loss of generality, the right part of the segment will be regarded alone.

Every household i, $i \in [1, N]$ has to trade off between a quantity of land S_i, which he purchases at a unitary land price $R(x)$ for his residence, and a part z_i of a composite good z, whose price is equal to 1. The assumption of exogenous quantities of land is now relaxed[2]. In that way, the impact of an environmental tool on the households' location behaviour may be considered.

The N households are employed by the firms. They all work. Each of them supplies one unit of labour which is paid at the exogenous wage w. Wage constitutes the entire income. A unitary travel cost parameter *t* is introduced. *t* is endured when commuting between residential location x, $x \in [r_{0,}, r_{\max}]$, and work place in the *CBD*. This place is assumed close to 0[3]. Every household living in x and distant from his working place in 0 endures daily proportional commuting costs equal to *2tx.* Initially, *t* is a private travel cost parameter (the cost of using a single occupant car for one kilometre travelled). The household program is given by:

$$\max_x U(S_i, z_i)$$

Subject to the constraint: $w - 2tx = R(x)S_i + z_i$ (1)

The utility function can equally be substitutable or complementary[4]. For the convenience of analysis, a complementary utility function is chosen so that:

$$U(S_i, z_i) = \min\{\delta S_i^{\gamma} \gamma z_i^{\delta}\}, \quad \gamma \geq 0, \delta \geq 0 \qquad (2)$$

As households are identical in terms of income and preferences, the equilibrium quantities of composite good and land are the same for all of them. Wherever they are located, they all reach the same level of equilibrium utility u^* and they consume the optimal quantities $S_i^* = u^*/\delta$ and $z_i^* = u^*/\gamma$. Combining the budget constraint (1), u^* and these optimal quantities, the individual bid-rent function of household i living in a location x is given by:

$$\psi(x, u^*, w, t) = \left[w - 2tx - \frac{u^*}{\gamma} \right] / \left[\frac{u^*}{\delta} \right] \qquad (3)$$

Equilibrium is presented in Figure 2. It depends on a bid-rent mechanism *à la* Alonso that initially takes place among households exclusively. Every household offers a bid land rent E_i to get the land. The bid-rent curve $E_i = \psi(x, u^*)$ denotes the highest price a household is willing to pay for one unit of land locating at a distance x from the *CBD*, $x \in [r_0, r_{\max}]$, which is different from his location at the equilibrium. We make the classical assumption of absentee landowners.

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[2] This assumption was previously relaxed in some papers (Anas and Kim, 1996, Lucas and Rossi-Hansberg, 2002). Ogawa and Fujita's (1980) elegant formal framework had however to be given up.
[3] Such a formulation is admitted in Alonso (1964).
[4] See Fujita (1989) for complete arguments of possible specifications.

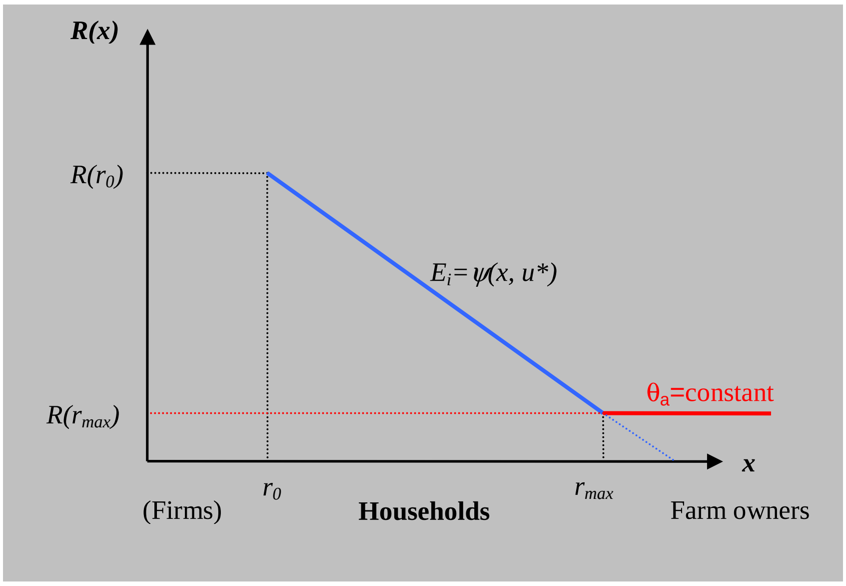

Figure 2 –Unitary land price curve (one type of agent)

Considering the given amount of commuting costs, the curve describes the share of income available for land after expenditures in the composite good. At every distance x, $x \in [r_0, r_{\max}]$, the household offering the highest bid land rent E_i occupies the land.

The unitary land rent curve $R(x) = \psi(x, u^*)$ decreases with the distance to the *CBD*. The specification retained for the utility function involves linear decreasing. The highest bid land rent is offered at $x = r_0$, interior boundary of the residential area. The lowest bid land rent is offered at $x = r_{\max}$, outer limit of the residential area, and city boundary. The more households decide to live close to the *CBD*, the higher is the unitary land price *R*. Nevertheless they endure lower commuting costs. Beyond the city boundary r_{max}, no more household offers bid land rents: farm owners occupy agricultural land, which they previously purchased at an agricultural land rent θ_a. This reservation price no longer depends on the distance to the *CBD*.

From (3), we have:

$$u^*(x, \psi, w, t) = \frac{\delta\gamma(w - 2tx)}{\delta + \gamma\psi} \tag{4.1}$$

The utility must be positive at every distance x so that:

$$w - 2tr_{\max} \geq 0 \tag{4.2}$$

Using (4.1) for $x = r_{\max}$:

$$u^*(r_{\max}, \theta_a, w, t) = \frac{\delta\gamma(w - 2tr_{\max})}{\delta + \gamma\theta_a} \tag{5}$$

We obtain the values for $S_i^* = u^*/\delta$, $n(x) = 1/S_i^* = \delta/u^*$ and $z_i^* = u^*/\gamma$.

2.2. Production equilibrium of firms

The firm is now introduced as a second type of agent. M firms j, $j \in [1, M]$ individually produce a part z_j of the composite good. The production level is obtained by combining a minimal quantity of labour L_j^* (paid at the unitary wage w) and an amount of land S_j. If $p_z = 1$, the firm program is given by:

$$\max_x \pi_j(z_j, x) = z_j - R(x)S_j - wL_j \qquad (6)$$

A production function with complementary inputs and constant returns to scale is preferred here[5] :

$$z_j = \min\{\alpha S_j, \beta L_j\} \qquad \alpha \geq 0, \beta \geq 0 \qquad (7)$$

The quantities of land and labour used by every firm are linked by the following relationship:

$$S_j = \frac{\beta}{\alpha} L_j \qquad (8.1)$$

The number of firms at every distance x, $x \in [0, r_0]$, is given by:

$$m(x) = {}^{1}\!/_{S_j} \qquad (8.2)$$

The properties of the production function clarify the accessibility function $A(x)$. Following Ogawa and Fujita (1980), local accessibility for a firm locating in x can be defined as spillover effects in which it takes advantage of another firm locating in y. Local accessibility may be defined in different ways; the monocentric equilibrium pattern that is considered here requires a linear form[6] such as :

$$a(x, y) = \lambda - \rho|x - y| \quad , \lambda \geq 0 , \rho \geq 0 \qquad (9.1)$$

For each firm locating in x, the aggregate accessibility is equal to the sum of local accessibilities; they constitute all the spillover effects a firm locating in x takes advantage of in the local neighbourhood. We assume that spillover effects constitute the complete agglomeration effects[7]. If local accessibility is linear, the aggregate accessibility is given by:

$$A(x) = \int_X [\lambda - \rho|x - y|]m(y)dy \qquad (9.2)$$

A high aggregate accessibility denotes intense contacts between close firms. These contacts work as a positive production externality which promotes firms' concentration within the *CBD*. Its strength is revealed by the value of the parameter ρ, which is an inverse measure of contact intensity linked to distance. If sales $p_z z_j$ are now included in $A(x)$, the program of firm j locating in x can be rewritten from (9.2):

$$\max_x \pi_j(x) = A(x) - R(x)S_j - wL_j \qquad (10)$$

Proof is presented in Appendix 2. As firms have, thanks to identical input quantities, to be indifferent to location within the *CBD*, they must reach the same level of equilibrium profit π^*: at every distance

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[5] It has been admitted that quantities of land and labour varying in the same direction were more realistic.

[6] An exponential form was studied by Ogawa and Fujita (1980) as well; it leads to multiple equilibria, some of them admitting several centres. See Imai (1982) for the proof of the monocentric pattern.

[7] See Jayet (1993) for a more complete description of spillover effects.

x, $x \in [0, r_0]$, the condition $\pi_j(x) = \pi_j(y)$ must apply. Adjustment then occurs through differentiated values of *A* in function of x. Then the long-term equilibrium condition imposes $\pi^* = 0$.

Every firm offers a bid land rent E_j to get the land. The bid-rent curve $E_j = \phi(x,0)$ denotes the highest price a firm reaching a zero-profit level is willing to pay for one unit of land locating at x. If an identical profit level is chosen for all firms in every place, then the individual bid-rent function of a firm j locating at x is given by:

$$\phi(x,0) = \left[A(x) - wL_{j_j}^*\right] / S_j^* \quad (11)$$

At every distance x, $x \in [0, r_0]$, the firm offering the highest bid land rent E_j occupies the land.

2.3. Model equilibrium without pollution

The monocentric model should be examined further. We now consider the tackling of households and firms on the markets. The urban monocentric pattern equilibrium with an endogenous centre is obtained simultaneously on three markets: on composite good, labour and land markets.

As the N households are identical in terms of income and preferences, they reach a single utility level u^* in equilibrium. u^* requires a single combination of quantities of land S_i^* and composite good z_i^*. In equilibrium, the total demand $N \times z_i^*$ is equal to the total production of the M firms $M \times z_j^*$. $z_i^* = z^*$ is assumed for the rest of the paper.

The equilibrium on the labour market requires:

$$N = M^* \times L_j^* \Leftrightarrow M^* = \frac{N}{L_j^*} \quad (12)$$

The number of firms M^* is determined. The N working households are employed by the M^* firms in equilibrium. Every firm individually seeks for an identical quantity of labour L_j^*.

The equilibrium on the land market requires three sets of conditions. The first one formalizes the graphical analysis in Figure 3:

$$\phi^*(x,0) \geq \psi^*(x,u^*)\text{ , } x \in [0, r_0] \quad (13.1)$$

$$\psi^*(x,u^*) \geq \phi^*(x,0)\text{ , } x \in [r_0, r_{\max}] \quad (13.2)$$

$$\psi^*(r_{\max}, u^*) = \theta_a \quad (13.3)$$

Within the *CBD*, firms offer a systematically higher bid land rent than households (13.1). In suburbs, households offer the highest bid land rent (13.2). At the boundary of the city, the bid land rent offered by households is equal to the agricultural land rent (13.3).

The double following condition is therefore required in equilibrium:

$$m(x) = \frac{1}{S_j^*}\text{ , } n(x) = 0\text{ , } x \in [0, r_0] \quad (14.1)$$

$$m(x) = 0\text{ , } n(x) = \frac{1}{S_i^*}\text{ , } x \in [r_0, r_{\max}] \quad (14.2)$$

The number of firms locating at a distance x from the *CBD* is inversely proportional to the quantity of land used, and the number of households in this area at every x is equal to 0. A conversed pattern can be noticed in the suburbs.

Lastly, all land in the whole city is used by firms and households, whereas land in the *CBD* is occupied by firms exclusively:

$$M^* S_j^* + N S_i^* = 2r_{\max} \tag{15.1}$$

$$M^* S_j^* = 2r_0 \tag{15.2}$$

These three sets of conditions denote that land use equilibrium depends on the widened individual bid-rent mechanism: at every distance from the *CBD*, households *and* firms now compete for land: at every distance x, the agent offering the highest bid land rent always sets up *(Figure 3).*

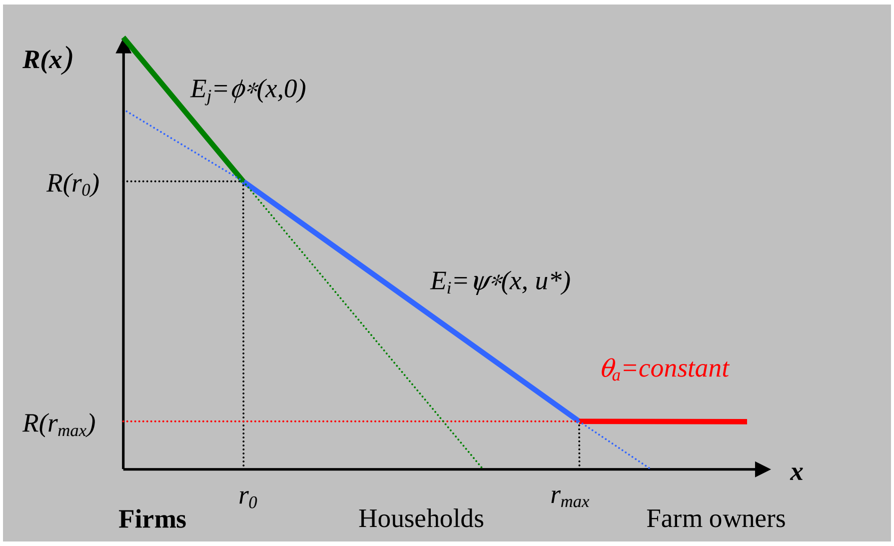

Figure 3 – Unitary land price curve (two types of agents)

The unitary land price curve is now given by: $R(x) = \max\{E_i, E_j\}, x \in [0, r_{\max}]$. Every part of the segment describes the upper cover of the bid land rent. In the *CBD,* $x \in [0, r_0]$, firms willing to group together in order to take advantage of agglomeration effects offer a systematically higher bid land rent than households. In the suburbs, $x \in [r_0, r_{\max}]$, firms are less interested in a relative dispersion that would isolate them. Firms offer a lower bid land rent and households set up, insofar as they estimate at the same time that commuting costs remain endurable. Beyond $r_{\max}$, farm owners keep occupying land.

The intermediate demand for land S_i^* and the number of households living in the suburbs, $x \in [r_0, r_{\max}]$, were previously obtained by using (13.3) to design residential equilibrium. Using (12), (15.1) and (15.2) afterwards, the equilibrium value of the (external) city boundary $r_{\max}(N, w, \theta_a, t)$ is:

$$r_{\max} = \frac{N(\beta(\delta + \gamma\theta_a) + \alpha\gamma w)}{2\alpha(\delta + \gamma(\theta_a + Nt))} \tag{16}$$

After firms and households compete on the land market, the demand functions for land $S_j^*(N, w, \theta_a, t)$ and for composite good $z^*(N, w, \theta_a, t)$ and the number of households become:

$$S_i^* = \frac{\gamma}{\alpha} \frac{(\alpha w - \beta N t)}{(\delta + \gamma(\theta_a + Nt))} \tag{17.1}$$

$$n(x) = \frac{1}{S_i^*} = \frac{\alpha}{\gamma}\frac{(\delta + \gamma(\theta_a + Nt))}{(\alpha w - \beta Nt)} \tag{17.2}$$

$$z^* = \frac{\delta(\alpha w - \beta Nt)}{\alpha(\delta + \gamma(\theta_a + Nt))} \tag{18}$$

The following condition must apply:

$$S_i^* \geq 0, z^* \geq 0 \Leftrightarrow \alpha w - \beta Nt \geq 0 \quad \forall t \geq 0 \tag{19}$$

The city boundary increases with total population, with the wage and with the agricultural land rent. In a linear city, the quantity of land used by household i does not depend on the distance to the *CBD*. It decreases with the size of population, but increases with the wage; it denotes the trade-off between demand for land and accessibility. The way the equilibrium quantity of composite good evolves with N and w is similar.

The value of the internal boundary r_0 as the limit of the *CBD* is provided by (15.2):

$$r_0 = \frac{\beta N}{2\alpha} \tag{20}$$

Solving (9.2) under the constraint (14.1) (that clarifies the monocentric pattern) gives the intermediate value of $A^*(x)$:

$$A^*(x) = \frac{-8\alpha^2\rho x^2 + 4\alpha\beta\rho Nx - \beta^2\rho N^2 + 4\alpha\beta\lambda N}{8\alpha\beta L_j^*} \tag{21.1}$$

$$4\alpha\beta N(\lambda + \rho x) - 8\alpha^2\rho x^2 - \beta^2\rho N^2 \geq 0 \tag{21.2}$$

Condition (21.2) must be filled. What happens in $x = r_0$ must be regarded to obtain a final expression of $A^*(x)$. The level of spillover effects in this specific point can be determined. Then the bid land rent offered by firms and households may be levelled out (*Figure 3*):

$$\frac{A^*(r_0) - wL_j^*}{S_j^*} = \frac{w - 2tr_0 - z^*}{S_i^*} \tag{22}$$

If S_i^*, z, S_j^*, $A^*(r_0)$ and r_0 are now replaced by their values, and if condition (23.2) applies, the demand function for labour $L^*(N, w, \theta_a, t)$ can be written:

$$L_j^* = \sqrt{\frac{N(4\alpha\lambda - \rho\beta N)}{8(\beta(\theta_a + Nt) + \alpha w)}} \tag{23.1}$$

$$L_j^* \in \Re \Leftrightarrow 4\alpha\lambda - \rho\beta N \geq 0 \tag{23.2}$$

The demand for labour decreases with *t*. If conditions (21.2) and (23.2) apply, L_j^* can then be substituted in $A^*(x)$:

$$A^*(x)=\frac{\sqrt{8(\beta(\theta_a+Nt)+\alpha w)}\left(4\alpha\beta N(\lambda+\rho x)-\beta^2\rho N^2-8\alpha^2\rho x^2\right)}{8\alpha\beta\sqrt{N(4\alpha\lambda-\rho\beta N)}} \tag{24.1}$$

Substituting the preceding value of L_j^* in equation (12.2), the demand function for land $S_j^*(N,w,\theta_a,t)$ becomes:

$$S_j^*=\frac{\beta}{\alpha}\sqrt{\frac{N(4\alpha\lambda-\rho\beta N)}{8(\beta(\theta_a+Nt)+\alpha w)}} \tag{25}$$

Condition (23.2) must apply. The demand for land by firm j in equilibrium decreases with *t*.

The urban monocentric pattern with an endogenous centre requires the sets of conditions (13.1) to (15.2). Alternative patterns are introduced in Appendix 1.

3. Working households' commuting trips, a source of pollution in the suburbs

3.1. The urban monocentric pattern equilibrium with pollution in a laisser-faire *situation*

In this section, pollution generated by working households' commuting trips is introduced. N polluting households are employed by M firms which are not supposed to generate pollution when producing. Their program remains unchanged. The specific impact of commuting trips is needed. In Section 4 the impact of an environmental tax on the adjusted residential location choices will be regarded[8]. All assumptions remain unchanged, except one:

Additional assumption 1: *because they commute daily by car, households generate pollutant emissions between an origin x as a residential place,* $x\in[r_0,r_{max}]$, *and a destination* x=0 *as a work place. This pollution D(x,0) (or level of environmental damage) is linear in relation to the entire geographic area in that:*

(i) The environmental damage increases with the distance travelled while commuting.

(ii) In a linear city, marginal damage is constant with the distance travelled while commuting.

In that way, pollution is homogenous in the entire urban area[9].

Additional definition 1: *because a number* n(x) *of households live at every distance x and commute every day to* x=0 *to work, the pollution function at every x can be specified as follows:*

$$D(x,0)=2kn(x)x\ \forall x\in[r_{0,}r_{max}] \tag{26.1}$$

k is a unitary pollution parameter. At every distance x, pollution constitutes the complete environmental damage to be corrected. Parameter *k* can be valued[10].

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[8] The damage generated by freight transport is not regarded, in particular inputs provided to the firms. There is no flow of the composite good between production and consumption places: it is made and sold in the same place. There are no specific household trips towards consumption places: their trips only concern commuting.

[9] Alternative specifications of the pollution function may be assumed: if congestion is admitted close to the *CBD* (increasing marginal emissions at the end of the trip), or if startings up of motor cars are considered (decreasing marginal emissions after the beginning of the trip). Through a linear form *k* will be immediately regarded as the marginal damage.

[10] For example: starting from the health cost recommendations linked to atmospheric pollution in the Boiteux report (2001), or from advocations for greenhouse gas emissions in the Quinet report (2009).

Replacing *n(x)* by its new value, the pollution function at every x, $x \in [r_0, r_{\max}]$, can be written:

$$D^*(x) = \frac{2k\alpha}{\gamma} \frac{(\delta + \gamma(\theta_a + Nt))}{(\alpha w - \beta Nt)} x \qquad (26.2)$$

3.2. Total damage and social cost of pollution

Substituting a large social cost (assuming that environmental damage is supported by the entire community/the city) for a private monetary cost exclusively endured by car-drivers is needed to introduce pollutant emissions.

<u>Additional definition 2:</u> *total social cost of pollution CST_1 can be defined in equilibrium as the sum of the private monetary cost CT_1 and the total damage DT_1:*

$$CST_1 = CT_1 + DT_1 \qquad (27)$$

<u>Additional definition 3</u>: *total damage can be defined in equilibrium as the sum of pollution functions generated by households at every location x from the CBD, $x \in [r_0, r_{\max}]$:*

$$DT_1 = \int_{r_0}^{r_{\max}} D^*(x)dx \qquad (28.1)$$

Replacing $D^*(x)$ by its value in (26.2):

$$DT_1 = \frac{kN^2\{\beta(2\delta + \gamma(2\theta_a + Nt) + \alpha\gamma w)\}}{4\alpha(\delta + \gamma(\theta_a + Nt))} \qquad (28.2)$$

Total damage is positive or zero for every $t \geq 0$.

<u>Additional definition 4:</u> *total commuting cost can be defined as the total damage if simply replacing parameter* k *by* t, *and if CT_1 is the sum of cost functions at every distance x from the CBD, $x \in [r_0, r_{\max}]$:*

$$CT_1 = \int_{r_0}^{r_{\max}} n(x)c_1(x)dx \qquad (29.1)$$

with $c_1(x) = 2tx, \ \forall x \in [r_0, r_{\max}]$ *(29.2)*

Replacing *n(x)* by its value:

$$CT_1 = \frac{tN^2\{\beta(2\delta + \gamma(2\theta_a + Nt) + \alpha\gamma w)\}}{4\alpha(\delta + \gamma(\theta_a + Nt))} \qquad (29.3)$$

Using (27), total social cost in a model with pollution in a *laisser-faire* situation is:

$$CST_1 = \frac{(k+t)N^2\{\beta(2\delta + \gamma(2\theta_a + Nt) + \alpha\gamma w)\}}{4\alpha(\delta + \gamma(\theta_a + Nt))} \qquad (30)$$

4. Pollution externalities *versus* agglomeration effects: is urban toll a relevant fiscal tool?

Internalizing environmental damage means substituting a generalized cost (including an additional environmental cost) for a private monetary cost. Time cost is not kept here in order to avoid an additive labour-leisure tradeoff. The environmental cost is internalized through a tax which is instituted by a local planner willing to maximize the social welfare through a drop in pollutant emissions. Why a tax rather than another environmental tool? Studies exist about tradable emissions quotas applicable to car-drivers (Raux, 2007). However, it will be considered here that environmental taxation is more tractable.

4.1. The urban monocentric pattern equilibrium with an eco-tax

The program of the non-polluting firm j is not affected by the eco-tax. Previous assumptions about household i are not affected, except for:

Additional assumption 2: *local planner decides to levy a tax in the form of a kilometric urban toll*[11] *whose amount is τ, $\tau>0$. The toll aims at internalizing the environmental damage generated by commuting trips. Working households have to pay from now on an amount of eco-tax per kilometre which is added to the previous one so that:*

$$c_2(x,0) = 2(t+\tau)x \qquad (31)$$

c_2 is considered as a generalized travel cost for household i using individual mode. The new demand for land in equilibrium and the number of households living at a distance x from the *CBD* become:

$$S_i^* = \frac{\gamma}{\alpha}\frac{(\alpha w - \beta N(t+\tau))}{(\delta + \gamma(\theta_a + N(t+\tau)))} \qquad (32.1)$$

$$n(x) = \frac{1}{S_i^*} = \frac{\alpha}{\gamma}\frac{(\delta + \gamma(\theta_a + N(t+\tau)))}{(\alpha w - \beta N(t+\tau))} \qquad (32.2)$$

Condition (19) is adjusted:

$$\alpha w - \beta N(t+\tau) \qquad (19')$$

Proposition 1*: after a kilometric urban toll has been achieved in a monocentric linear city with an endogenous centre, the equilibrium demand for land by households at every distance x from the* CBD *is lower than in a* laisser-faire *situation, and the number of households increases everywhere in the city. This outcome applies for every $(t+\tau)$, that is to say whatever the initial amount of unitary travel cost, or whatever the amount of toll. Condition (19') and the household budget constraint must apply.*

This property depends on the particular forms that were chosen for the utility and the production functions. The identical quantity of land used by every household remains independent of x. In the trade-off between demand for land and accessibility, the toll reduces the land used by households in equilibrium. Indeed, the share of budget available for housing is lessened. The equilibrium quantity of composite good is lessened as well.

The city boundary becomes:

$$r_{\max} = \frac{N(\beta(\delta + \gamma\theta_a) + \alpha\gamma w)}{2\alpha(\delta + \gamma(\theta_a + N(t+\tau)))} \qquad (33)$$

Proposition 2*: after a kilometric urban toll has been achieved in a monocentric linear city with an endogenous centre, the city is more compact in consequence of the relocation of some households*

[11] For example: a kilometric tax applied to a network of urban motorways.

closer to the CBD*. The size of the* CBD *remains unchanged. This outcome applies for every (t+τ), that is to say whatever the initial amount of unitary travel cost, or whatever the amount of toll. Condition (19') and the household budget constraint must apply.*

The aggregate accessibility function $A^*(x)$ is now given by:

$$A^*(x) = \frac{\sqrt{8(\beta(\theta_a + N(t+\tau)) + \alpha w)}\left(4\alpha\beta N(\lambda + \rho x) - \beta^2 \rho N^2 - 8\alpha^2 \rho x^2\right)}{8\alpha\beta\sqrt{N(4\alpha\lambda - \rho\beta N)}} \quad (34)$$

<u>Proposition 3</u>*:* *after a kilometric urban toll has been achieved in a monocentric linear city with an endogenous centre, the agglomeration effects which firms take advantage of within the* CBD *in equilibrium are <u>strengthened</u>. This outcome applies for every (t+τ), that is to say whatever the initial amount of unitary travel cost, or whatever the amount of toll. Conditions (23.2) and (21.2) must apply.*

In particular, condition (21.2) must apply on the entire range of values of x between 0 and r_0. This outcome depends on the sensitivity of each production input to *t*. A raise in the unitary travel cost causes a joined fall in equilibrium in the demand for labour and for land. As equation (20) specifies that the size of the *CBD* remains unchanged, the number of firms after the toll is higher. As the aggregate accessibility function depends on the number of firms or, identically, on the inverse of the demand for land, then, all things being equal, the level of spillover effects after the toll is higher. This outcome still depends on the particular form chosen for the production function.

4.2. The improvement of social welfare after the urban toll

<u>Additional definition 5:</u> *the social welfare function W is given by:*

$$W = \left(N \times U(z_i, S_i) - DT_1\right) \quad (35)$$

As profit is zero in equilibrium, *W* does not depend on the program of the firm. The local planner maximizes the social welfare function through an eco-tax aimed at achieving a second best optimum, or, alternately, he minimizes the total social cost expression in (30). His program is:

$$\min_{t+\tau}\left[CST_1 = \frac{(k + (t+\tau))N^2\{\beta(2\delta + \gamma(2\theta_a + N(t+\tau)) + \alpha\gamma w)\}}{4\alpha(\delta + \gamma(\theta_a + N(t+\tau)))}\right]$$

Equating the derivative in relation to *(t+τ)* to zero gives the optimal positive value (36.1), if conditions (36.2) and (36.3) hold:

$$(t+\tau)^0 = \frac{-\beta\gamma N(\delta + \gamma\theta_a) + \sqrt{\beta\gamma^2 N^2(\gamma(kN - \theta_a) - \delta)(\beta(\delta + \gamma\theta_a) + \alpha\gamma w)}}{\beta\gamma^2 N^2} \quad (36.1)$$

$$\beta\gamma^2 N^2(\gamma(kN - \theta_a) - \delta)(\beta(\delta + \gamma\theta_a) + \alpha\gamma w) \geq 0 \quad (36.2)$$

$$\sqrt{\beta\gamma^2 N^2(\gamma(kN - \theta_a) - \delta)(\beta(\delta + \gamma\theta_a) + \alpha\gamma w)} \geq \beta\gamma N(\delta + \gamma\theta_a) \quad (36.3)$$

The optimal value of the ecotax rises with the unitary pollution parameter *k*.

4.3. Numerical example: the impact of a 10 % raise in t *a 100 000 working household city*

The aim of this paragraph is to illustrate the way a convenient amount of urban toll may be applied to a medium-sized city, and the way it positively impacts the level of agglomeration effects in the *CBD*. Some values are suggested for exogenous variables and parameters (*table 1*). The sensitivity of those values is tested in Appendix 4.

Variable/ parameter	**Description**	**Benchmark value**
N	Number of working households *(units)*	100 000
θa	Agricultural land rent *(€ per m² and per day)*	0,25
w	Daily wage paid by firm j to household i *(€ per day)*	100
α	Share of factor S_j in the production function of firm j *(units)*	100
β	Share of factor L_j in the production function of firm j *(units)*	0.5
δ	Share of commodity S_i in the utility function *(units)*	0.1
γ	Share of commodity z_i in the utility function *(units)*	1.05
λ	Constant of accessibility *(units)*	20
ρ	Contact intensity linked to distance *(units)*	0.1
k	Unitary pollution parameter *(€ par 100 cars.km)*	0.5
t	Unitary travel cost *(€ par 100 cars.km)*	50

Table 1 – Benchmark values of exogenous variables and parameters

The value retained for N reflects a medium-sized city. The agricultural land rent θ_a is consistent with plots on the edge of medium-sized cities which are not building land yet. All households work 20 days a month. The daily wage is set at 100 € per day worked. The unitary pollution parameter is valued from the cost of one tonne of CO_2 (32 $€_{2008}$) applied to cars emitting 154 grams per kilometre on average[12]. Lastly the unitary travel cost before pollution pricing is set at 50 €/ 100 cars.km[13]. The previous constraints still apply:

$$w - 2(t+\tau)r_{\max} \geq 0 \qquad (4.2')$$

$$\alpha w - \beta N(t+\tau) \geq 0 \qquad (19')$$

$$4\alpha\beta N(\lambda + \rho x) - \beta^2 \rho N^2 + 8\alpha^2 \rho x^2 \geq 0 \qquad (21.2)$$

$$4\alpha\lambda - \rho\beta N \geq 0 \qquad (23.2)$$

Solving (36.1) gives a quasi-generalized travel cost *(t+τ)* = 55 €/ 100 cars.km. This is as if the local planner would raise commuting costs by 10 % through a kilometric urban toll in order to limit pollutant emissions. Impacts are summarized in table 2.1.

Variable	**Laisser-faire *situation***	***Optimal policy***
(t+τ)(€/ 100 cars.km)	***50***	***55***
CST (€/ day)	*2 514 040*	*2 513 970*
DT (€/ day)	*24 892*	*22 531*
A(t,r0/2)(€/ day)*	*11 241*	*11 242*
M (units)*	*1635*	*1635*
r_{max} *(km)*	*99,3*	*89,9*
r_0*(km)*	*0,25*	*0,25*

Table 2.1 – Impact of a toll raising the travel cost by 10 % (N=100 000)

[12] If the local policy aims at regulating atmospheric rather than greenhouse gas emissions, then, according to the recommendations of the Boiteux report for health costs in a concentrated urban area (2,9 $€_{2000}$ pour 100 cars.km), this cost would be multiplied ten times. Moreover we checked that outcomes would not be strongly affected.

[13] This amount is consistent with fiscal administration scales relative to travel expense repayment in France.

In a 100 000 working household city, a 10 % raise in commuting costs by implementing a kilometric urban toll leads to a drop of 9,5 % in pollutant emissions, whereas the city size contracts from 9,5 %[14]. The city boundary may seem distant. It must actually be regarded as the urban area boundary, which is much larger than the city itself: it is the largest attraction area of working households. That means concretely that one or two districts which were set inside the urban area in equilibrium are no longer there after levying the urban toll (attraction area is reduced by 9,4 km). Working households who lived there are now located in another district closer to the *CBD*.

The additive commuting costs generated by the toll are evaluated at about 2000 € a day (as a raise in the total cost CT_1). These additive costs have to be divided between 100 000 working households, which works out as 0,40 € per household a month. This very low number must be interpreted as a net outcome: precisely as the difference between the additive costs paid by quite heavily imposed "static" households, and the additive costs avoided by "dynamic" households who chose to get closer to the *CBD* to limit their commuting expenditure. In the case of the abatement of atmospheric pollutant emissions, these additive costs would be rather higher.

Lastly, the impact of spillover effects through the accessibility function is positive. Indeed the value of $A^*(r_0/2)$ (half distance between $x=0$ and $x=r_0$) rises by 11 240,90 € a day to 11 242,40 € in the centre of the *CBD*. This increase is however extremely low in percentage (+0,01 %). In the case of the abatement of atmospheric pollutant emissions, the benefits would be rather higher. As the number of workers employed by every firm hardly varies after the toll, the number of firms M (rounded off) remains unchanged in a *laisser-faire* or in a regulation situation.

If the local planner does not know the optimal tax amount, he could however implement an environmental policy with concrete benefits in terms of pollutant emissions control (*table 2.2*).

Variable	**Laisser-faire situation**	***Moderate policy***	***Optimal policy***	***Voluntarist policy***
(t+τ)(€/ 100 cars.km)	***50***	***52,50 (+5 %)***	***55 (+10 %)***	***60 (+20 %)***
CST (€/ day)	*2 514 040*	*2 513 990*	*2 513 970*	*2 514 020*
DT (€/ day)	*24 892*	*23 717*	*22 531*	*20 777*
A(t,r0/2)(€/ day)*	*11 240,9*	*11 241,6*	*11 242,4*	*11 243,7*
*M *(units)*	*1635*	*1635*	*1635*	*1635*
r_{max} *(km)*	*99,3*	*94,6*	*89,9*	*82,9*
r_0*(km)*	*0,25*	*0,25*	*0,25*	*0,25*

Table 2.2 – Compared impacts of different amounts of toll (N=100 000)

Thus, twice as low pollution pricing, 5 % instead of 10 ("moderate policy") also leads to a linked and quasi-proportional fall in pollutant emissions and in the city boundary (4,7 % instead of -9,5). Conversely, in a twice as high pricing scheme, 20 % instead of 10 ("voluntarist policy"), the benefits in terms of the drop in pollutant emissions keep working, but less than expected (-16,5 compared with the equilibrium, against -19,0 expected). Though total cost including the additive commuting costs keeps increasing, and makes global utility of households worse. The impact on accessibility is slightly higher than in the optimal pollution pricing scheme, but it nevertheless remains very low.

Two alternative examples are introduced in Appendix 3.1 and 3.2.

5. Conclusion

In this paper based on urban economics, the impact of a kilometric urban toll in a spatial framework was regarded. Indeed any local government should be concerned with internalizing external effects of

[14] The static model does not take into account the time households need to relocate closer to the *CBD*.

pollution generated by working households' commuting trips. In order to take into account agglomeration effects as a positive production externality, households and firms locate within the city in an endogenous manner.

We started from a particular equilibrium pattern (the monocentric city) which describes big European cities. In consequence of agglomeration effects that enhance firms to offer higher bid land rents to group together, all jobs are settled in the *CBD*. The suburbs are occupied by households. When they move away, households seek larger quantities of land. They are incited to by historically low travel costs. Such a pattern is one ground for urban sprawl that causes additional pollutant emissions.

Pollutant emissions generated by households' commuting trips were introduced in this monocentric city. Emissions were then internalized, implementing a toll, whose optimal value was calculated. **We argued that starting from a monocentric equilibrium pattern, implementing a kilometric urban toll to abate pollutant emissions reduces the demand for land by households, leading to a compact city with lower commuting trips and emissions.** Numerical examples confirmed that **urban sprawl may be countered as a side-effect when a pollution abatement policy is achieved**. In a 100 000 working household city, the model predicts that a raise of commuting costs by 10 % by implementing an urban toll leads to a drop in both emissions and urban sprawl of 9,5 %, exclusively because the most distant households relocate closer to the *CBD*.

As for firms, a more unexpected outcome emerged: in consequence of a higher number of firms individually using smaller quantities of land, **the kilometric urban toll, whatever the tax amount, strengthens agglomeration effects which firms take advantage of within the *CBD*.** However, a numerical example illustrates that such an impact is limited.

These outcomes seem to confirm that environmental tools are relevant in local contexts of pollution abatement, even when a second source of distortion works. Later work will have to consider public transport as an alternative solution to individual cars. It would be of some interest to use the same analytical framework and to check if social welfare could be improved when households change their transportation mode, and if urban sprawl still could be controlled. However, a first interesting extension would be to check the robustness of the positive impact of urban tolls on agglomeration effects when firms partially pay commuting costs.

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Appendix 1

When conditions (13.1) to (15.2) do not apply any longer, the urban equilibrium pattern is not monocentric. It can be completely integrated, that is to say without any land use specialization (households and firms mix within the entire urban area), or incompletely integrated: some households living in the *CBD* are surrounded by firms settled in inner suburbs, which are surrounded by other households living in outer suburbs. These alternative patterns depend on threshold values for the unitary travel cost t and for the contact intensity linked to distance ρ (Fujita and Thisse, 2003, p.254).

Why did we not keep one or the other alternative patterns of urban economics with endogenous centre to introduce pollutant emissions? The completely integrated pattern does not lead to a truly spatial analysis, because there is no land use specialization. In addition, this scheme is associated with high values for the unitary travel cost combined with weak interactions between firms: agglomeration effects are not very active. This pattern does not respond to the realism expected when a sufficient level of agglomeration effects occurs.

The incompletely integrated pattern is more interesting. Households' location is assumed in the *CBD* and in suburbs at the same time. This would make this pattern realistic in regard of large European cities. Indeed, European households, taking into account their income, live in the *CBD* where land use is costly *and* in suburbs where it is more affordable. Firms occupy an intermediate area, often near urban motorways.

In order to simplify the analysis, we chose the monocentric pattern and considered the number of households living in the *CBD* negligible compared with those living in suburbs. In addition, the ratio t/ρ is the highest of the three possible patterns. This combination: relatively low travel costs/ relatively high agglomeration effects is of some interest. Correcting pollutant emissions in that pattern then permitted to check that, after an urban toll was achieved, urban sprawl through agglomeration effects should be slowed down, even inversed.

Appendix 2

The content of this appendix is integrally drawn from Fujita and Thisse (2003, p.241 -243). All M firms produce the same amount z_j of the z commodity sold at price $p_z = 1$ using the same technology. Each firm needs a quantity of land S_j and a quantity of labour L_j for production. The quantity of outputs z_j produced by the firm also depends on the quantity of information provided by the other firms within the city.

Firms are symmetric but different through the type of information they own. Each firm will then be willing to communicate actively with all other firms. The intensity of communications is measured by the number of contacts (e.g., the number of face to face contacts), and each firm chooses its optimal number of contacts. If $\varphi(x, y)$ is the number of contacts a firm located in x chooses to have with a firm located in y, then the total contribution of these contacts to the firm's sales will be $V[\varphi(x, y)]$. We suppose that this contribution is the same for everyone in order to respect the assumption of symmetry.

Communication between firms requires time from the two parts involved with the exchange. This time is needed for organisation, stocking, analysis and communication of information. When a firm located in x gets information from a firm located in y, it must endure a cost $c'(x, y)$ by unity of contact. This cost is supposed to depend on the location of each of the two firms. At the same time, the firm located in y will bear a cost c'' which is independent of location. For example, when the boss of a firm located in x calls the boss of a firm located in y, he imposes a cost to his correspondent, because the latest gives him part of his time, but this cost does not depend on distance between firms. That means that every firm will endure an additive cost induced by the activity of communication of the other firms.

$m(y)$ is the density of firms located in y, and $\varphi(x, y)$ is the number of contacts a firm located in x chooses to have with every firm located in y. The sales of a firm located in x are then given by:

$$p_z z_j = \int \{V[\varphi(x, y)]\} m(y) dy \text{ with } p_z = 1$$

and its profit, by:

$$\begin{aligned}\pi(x) &= z_j - \int [c'(x, y)\varphi(x, y) + c''\varphi(y, x)] m(y) dy - R(x) S_j - wL_j \\ &= \int \{V[\varphi(x, y)] - c'(x, y)\varphi(x, y) - c''\varphi(y, x)\} m(y) dy - R(x) S_j - wL_j\end{aligned}$$

Considering spatial location and levels of contacts of other firms fixed, each firm chooses the location and the level of contacts $\varphi(x, y)$ which maximize its profit.

Choosing $\varphi(x,y)$ such as $V[\varphi(x,y)]-c'(x,y)\varphi(x,y)$ is maximized, the optimal level of contacts a firm located in x will have with any other firm located in y may be determined, independently of the complete location of firms. If $c'(x,y)=c'(y,x)$, then the communication process between firms is symmetric, so that the optimal level of contacts between each pair of firms is the same for everyone: $\varphi^*(x,y)=\varphi^*(y,x)$.

Local accessibility of each pair of locations (x,y) is given by:

$$a(x,y)=V[\varphi^*(x,y)]-[c'(x,y)+c'']\varphi^*(x,y)$$

The profit function is then:

$$\pi(x)=A(x)-R(x)S_j-wL_j$$

where:

$$A(x)=\int a(x,y)m(y)dy$$
$$=\int\{V[\varphi^*(x,y)]-[c'(x,y)+c'']\varphi^*(x,y)\}m(y)dy$$

Notice that $a(x,y)$ may otherwise be interpreted as a spillover effect which will benefit to a firm located in x from a firm located in y. In that case, $A(x)$ may be considered as a « gap function » delaying the reception of information. It appears like a spatial externality. The amount of information received by a firm is exogenous; it still depends on the location of the firm relatively to all others'.

If associated with previous profit function, the individual bid-rent function of a firm located in x is:

$$\phi(x,\pi)=\frac{A(x)-wL_j-\pi}{S_j}$$

Appendix 3.1

Two alternative schemes aimed at illustrating relative efficiency of an urban toll applied to different sized cities are now suggested. Other variables and parameters were adapted. We seek to measure the impact of an unchanged optimal urban toll (10 % of the unitary travel cost) on the level of pollutant emissions. The impact on urban sprawl will be considered as a side-effect. The impact on spillover effects through the accessibility function is regarded as well.

The first variant applies to a small 20 000 working household city (*table 3.1*). Outcomes associated with pricing discrepancies are regarded as well.

Variable	**Laisser-faire situation**	***Moderate policy***	***Optimal policy***	***Voluntarist policy***
(t+τ)(€/ 100 cars.km)	***50***	***52,50 (+5 %)***	***55 (+10 %)***	***60 (+20 %)***
CST (€/ day)	*376 112*	*376 108*	*376 107*	*376 112*
DT (€/ day)	*3724*	*3548,19*	*3407,17*	*3108,36*
A(t,r0/2)(€/ day)*	*2906,67*	*2906,77*	*2906,86*	*2907,06*
M (units)	*775*	*775*	*775*	*775*
r_{max} *(km)*	*74,4*	*70,9*	*68,0*	*62,1*
r_0*(km)*	*0,1*	*0,1*	*0,1*	*0,1*

Table 3.1- Compared impacts of different amounts of toll (N=20 000)

The amounts of unitary travel cost and unitary pollution parameter remain unchanged (50 and 0,5 €/ 100 cars.km. In order to simulate a relaxed pressure on building lands around the city, the agricultural land rent is lowered to 1 % of the benchmark scheme (θ_a =0,0025 €/ m²/ day worked). The income of working households is reduced by 25 % to 75 €/ day worked. In order to have a number of firms tending to decrease within the *CBD,* the constant of accessibility λ is lowered from 20 to 10. However, the contact intensity linked to distance parameter ρ remains unchanged.

When (36.1) is solved, this new set of variables leads to an unchanged optimal value ($t+\tau$) =55 €/ 100 cars.km. The benefits are a little lower (in percentage) than those achieved in a city 5 times as big: (-8,5 % for pollutant emissions and for -8,6 for urban sprawl). In the case of lower or higher tax levels compared with the optimal amount of toll, the benefits obtained are strictly comparable with the benchmark scheme. The impact of the urban toll on agglomeration effects remains positive, yet still weak (in particular in absolute terms).

Appendix 3.2

The second variant deals with a large 500 000 working household city (*table 3.2*).

Variable	**Laisser-faire situation**	***Moderate policy***	***Optimal policy***	***Voluntarist policy***
(t+τ)(€/ 100 cars.km)	***50***	***52,50 (+5 %)***	***55 (+10 %)***	***60 (+20 %)***
CST (€/ day)	*15 764 100*	*15 763 600*	*15 763 500*	*15 764 000*
DT (€/ day)	*156 080*	*148 714*	*142 282*	*130 281*
A(t,r0/2)(€/ day)*	*42 010,5*	*42 014,6*	*42 018,7*	*42 027,2*
M (units)	*2241*	*2241*	*2241*	*2242*
r_{max} *(km)*	*124,3*	*118,5*	*113,3*	*103,7*
r_0*(km)*	*0,5*	*0,5*	*0,5*	*0,5*

Table 3.2 - Compared impacts of different amounts of toll (N= 500 000)

The amounts of unitary travel cost and unitary pollution parameters remain unchanged. The agricultural land rent is now multiplied by five compared with the benchmark scheme (θ_a =1,25 €/ day worked). w is paid to more qualified workers in average and raised by 25 % to 125 €/ day worked. In order to have a number of firms tending to rise within the *CBD,* the constant of accessibility λ is raised from 20 to 50. The contact intensity linked to distance parameter ρ remains unchanged.

This new set of variables leads to an unchanged optimal value ($t+\tau$) =55 €/ 100 cars.km. The benefits are a little lower (in percentage) than those achieved in the benchmark city: (-8,8 % for pollutant emissions and for urban sprawl). In the case of lower or a higher tax levels compared with the optimal amount of toll, the benefits obtained are comparable (in percentage) with the benchmark scheme. The impact of the urban toll on agglomeration effects remains positive, yet still very weak.

Appendix 4

In order to help the local planner to determine the exact optimal toll level, we seek to identify the parameters whose variation strongly affects the quasi-generalized cost of travel $(t+\tau)$, therefore the value of the optimal tax level τ . The analytical expression of τ indicates that the constant of accessibility λ as well as the contact intensity linked to distance ρ does not have any impact on the unitary travel cost, therefore on the tax level to be implemented.

We thus study the impacts of variations of the five other parameters $\alpha, \beta, \delta, \gamma$ and k on the value of $(t+\tau)$. As in the numerical example, we assume that the initial private travel cost is fixed to 50 €/ 100 cars.km (or 0,0005 €/ m). Then we analyze to what extent a lower or a higher amount for each of these five parameters may lead the local planner to move away from the optimal tax level. The values for exogenous variables and parameters that do not vary are the ones fixed in the numerical example.

1. *The impact of a variation of the intensity of factor* S_j *in the production function*

We first consider to what extent a wrong estimation of the intensity in land consumption by firms (parameter α) may lead the local planner to levy a wrong tax level. We recall that all firms use the same production technology ($\alpha_j = \alpha_{j'}, j \neq j'$). In addition, this case is based on the simultaneous use of a relatively high level of labour (parameter β). Indeed, the production function being complementary, the quantity of the less used input determines the quantity used for the other.

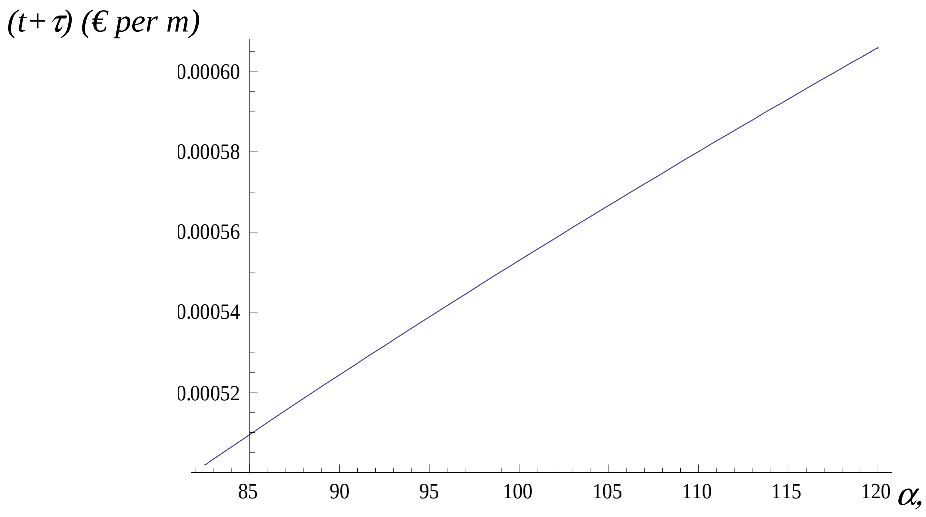

Figure 4.1 - Impact of a variation of the intensity of factor S_j in the production function

Under this condition, we saw that the optimal value for α was 100. It leads to a 10 % tax level for a quasi-generalized travel cost of 55 €/ 100 cars.km. Underestimating by 10 % the mean intensity in land consumption by firms (α=90) leads to a quasi-generalized travel cost of 52,40 €/ 100 cars.km (under taxation of the initial travel cost by 4,8 % instead of 10). Conversely, overestimating by 10 % this intensity (α=110) leads to an over taxation to 58,01 €/ 100 cars.km (over taxation by 16 % instead of 10). The extent of these tax errors remains in the range of the benchmark scheme. Their impacts on the total social cost, the total pollution, the city boundary and the spillover effects may be valued.

2. *The impact of a variation of the intensity of factor* L_j *in the production function*

Contrary to α , the optimal tax level decreases with the intensity in labour input β by firms. An estimated intensity higher than the real value leads to a lower optimal toll amount. The previous reservation still holds: there still must be a sufficiently high intensity in land so that it is the quantity of labour used that determines the optimum as the less used input.

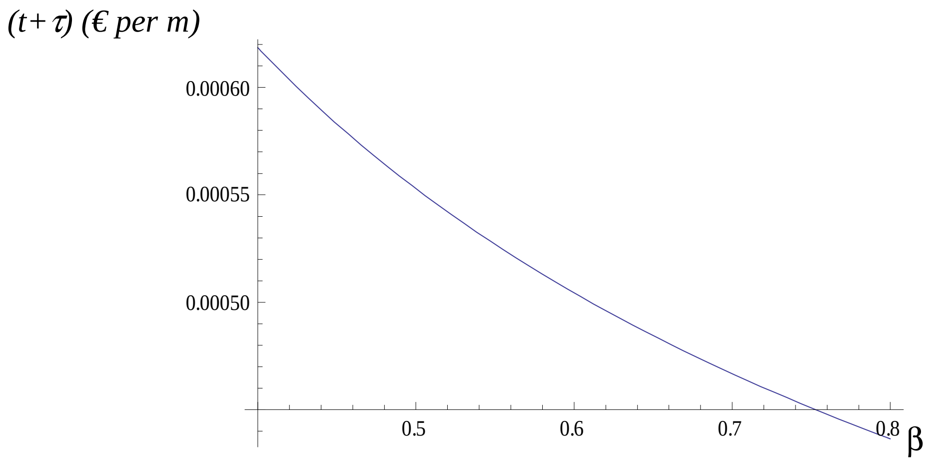

Figure 4.2 - Impact of a variation of the intensity of factor L_j in the production function

We saw that the optimal value for β was 0,5. It leads to a 10 % tax level for a quasi-generalized travel cost of 55 €/ 100 cars.km. Underestimating by 10 % the mean intensity in labour (β=0,45) leads to a quasi-generalized travel cost of 58,30 €/ 100 cars.km (over taxation of the initial travel cost by 16,6 % instead of 10). On the contrary, overestimating by 10 % this intensity (β=0,55) leads to an under taxation to 52,70 €/ 100 cars.km (under taxation by 5,4 % instead of 10). The extent of these tax errors remains in the range of the benchmark scheme.

3. The impact of a variation of the share of the commodity S_i *in the utility function*

We now consider what happens from the consumption structure side of the household. In other words: does an overestimated preference for land strongly affect the optimal tax level? We thus examine the respective values for parameters δ and γ. We start with δ (Share of commodity S_i in the utility function). The utility function being complementary goods, previous conditions still apply.

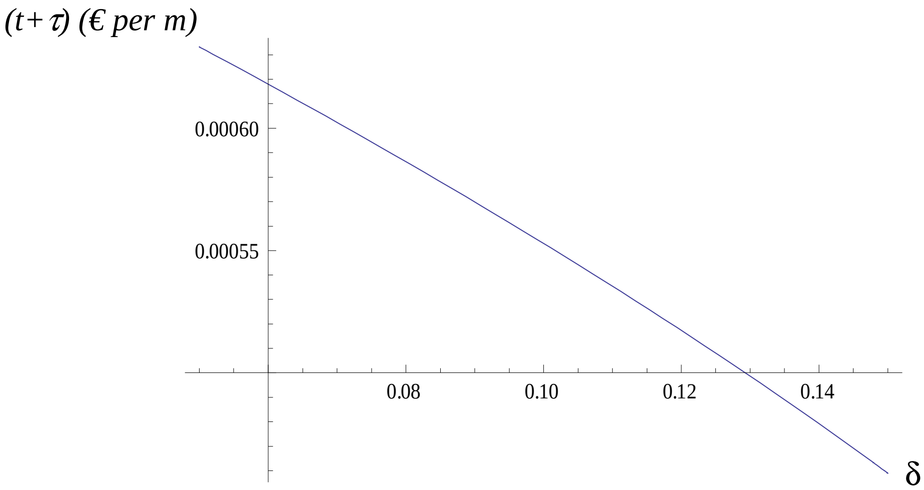

Figure 4.3 - Impact of a variation of the share of the commodity S_i *in the utility function*

The optimal value for δ was 0,1. The impact of δ on the tax level is decreasing. Underestimating δ by 10 % (δ=0,09) leads to a higher quasi-generalized travel cost of 56,99 €/ 100 cars.km (over taxation of the initial travel cost by 14 % instead of 10). On the contrary, overestimating by 10 % this intensity (δ=0,11) leads to an under taxation to 53,54 €/ 100 cars.km (under taxation by 7 % instead of 10). The elasticity of δ is thus lower than for the two firms' parameters. Estimation errors here have half as much impact than for firms.

4. The impact of a variation of the share of the land z_i *in the utility function*

The same analysis is done for the parameter γ (share of the composite good in the utility function).

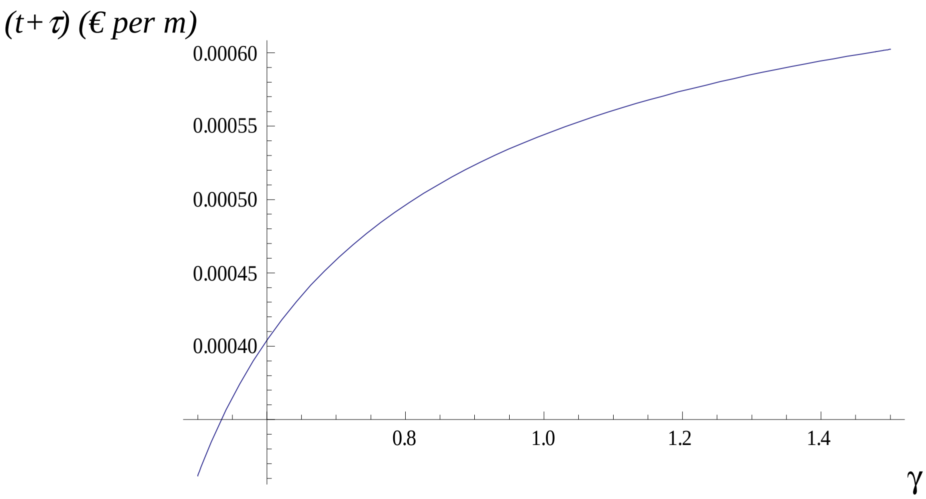

Figure 4.4 - Impact of a variation of the share of the land z_i *in the utility function*

The optimal value for γ was 1,05. The impact of γ on the tax level is increasing. Underestimating γ by 10 % (γ=0,945) leads to a quasi-generalized travel cost of 53,34 €/ 100 cars.km (under taxation of the initial travel cost by 6,6 % instead of 10). On the contrary, overestimating by 10 % this intensity (γ=0,955) leads to an over taxation to 56,83 €/ 100 cars.km (over taxation by 13,6 % instead of 10). The elasticity of γ is lower than for the two firms' parameters. In absolute value, it remains comparable to δ .

5. The impact of the variation of the unitary pollution parameter k

We lastly see how a wrong estimation for the unitary pollution parameter *k* may lead to a false estimation of the optimal tax level.

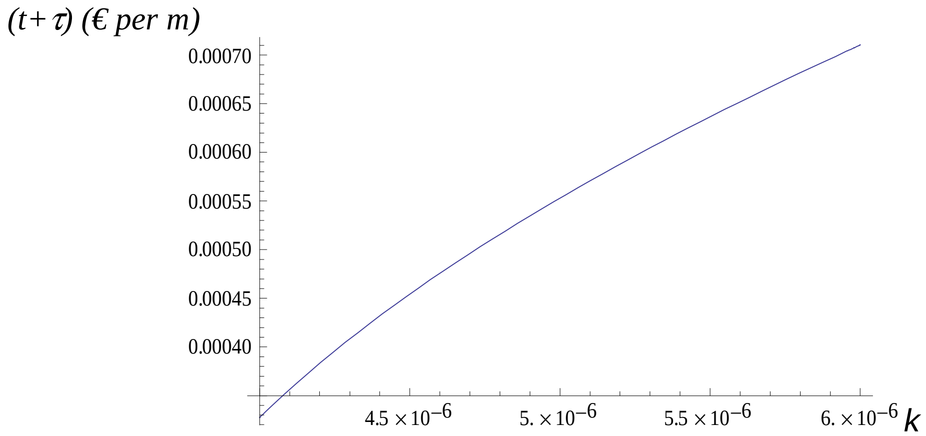

Figure 4.5 - Impact of the variation of the unitary pollution parameter k

The optimal tax level logically rises with *k*. A more valued pollution requires a higher tax to be corrected. Underestimating *k* by 10 % (*k*=0,45) leads to a quasi-generalized travel cost of 45,40 €/ 100 cars.km. **It is less than the initial travel cost**. On the contrary, overestimating *k* by 10 % (*k*=0,55) leads to a very high over taxation to 63,65 €/ 100 cars.km (over taxation by 27,4 % instead of 10). **In that scheme, the public policy may appear strongly inefficient.**

Major lessons:

- From the firm's viewpoint, parameters α and β play in opposite directions, although in comparable proportions. Their individual impacts around the optimum are high (with an absolute value of the elasticity of the quasi-generalized travel cost close to 0,5). The shape for the production function acts in favour of the local planner. Even if the two values are wrong, only the one associated with the lowest quantity of factor used will have an impact on the optimum, because it simultaneously determines the amount of the other factor. In addition, the extent of the impact of the two parameters being comparable, there is no additional risk to wrongly estimate the "bad" parameter.

- From the household's viewpoint, parameters δ and γ play in opposite directions and in comparable proportions as well. The optimal tax level is less sensitive to the consumption structure than to the production technology. Their individual impacts around the optimum are less high than for firms (with an absolute value of the elasticity of the quasi-generalized travel cost close to 0,3). The shape for the utility function acts in favour of the local planner as well. Even if the two values are wrong, only the one associated with the lowest quantity of factor used will have an impact on the optimum, because it determines the amount of the other factor. In addition, the extent of the impact of the two parameters being comparable, there is no additional risk to wrongly estimate the "bad" parameter.

- Lastly, the unitary pollution parameter *k* strongly affects the tax level upwards. Errors in estimation around the optimum are far more costly here for the local planner than in previous parameters (elasticity close to 1,5). The risk of particularly inefficient public policies is thus high. However, the estimation of this parameter is probably easier than for the previous ones, at least for carbon dioxide (Boiteux 2001, Quinet 2009).